

$D5$ -brane type I superstring background fields in terms of type IIB ones by canonical method and T-duality approach *

B. Nikolić [†] and B. Sazdović [‡]

Institute of Physics, 11001 Belgrade, P.O.Box 57, Serbia

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Abstract

We consider type IIB superstring theory with embedded $D5$ -brane and choose boundary conditions which preserve half of the initial supersymmetry. In the canonical approach that we use, boundary conditions are treated as canonical constraints. The effective theory, obtained from the initial one on the solution of boundary conditions, has the form of the type I superstring theory with embedded $D5$ -brane. We obtain the expressions for $D5$ -brane background fields of type I theory in terms of the $D5$ -brane background fields of type IIB theory. We show that beside known Ω even fields, they contain squares of Ω odd ones, where Ω is world-sheet parity transformation, $\Omega : \sigma \rightarrow -\sigma$. We relate result of this paper and the results of [1] using T-dualities along four directions orthogonal to $D5$ -brane.

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1 Introduction

It is a known fact that states of the type IIB superstring theory even under world-sheet parity projection $\Omega : \sigma \rightarrow -\sigma$ correspond to the states of type I superstring theory [2]. More precisely, the states, which correspond to the background fields: graviton $G_{\mu\nu}$ and dilaton Φ from NS-NS sector, the sum of two same chirality gravitinos $\psi_{+\mu}^\alpha = \psi_\mu^\alpha + \bar{\psi}_\mu^\alpha$ and dilatinos $\lambda_+^\alpha = \lambda^\alpha + \bar{\lambda}^\alpha$ from NS-R sector and two rank antisymmetric tensor $A_{(2)}$

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[†]e-mail address: bnikolic@ipb.ac.rs

[‡]e-mail address: sazdovic@ipb.ac.rs

from R-R sector, survive this projection. The states that are odd under Ω transformation: antisymmetric tensor $B_{\mu\nu}$ from NS-NS sector, difference of two gravitinos $\psi_{-\mu}^\alpha = \psi_\mu^\alpha - \bar{\psi}_\mu^\alpha$ and dilatinos $\lambda_-^\alpha = \lambda^\alpha - \bar{\lambda}^\alpha$ from NS-R sector, and scalar $A_{(0)}$ and four rank antisymmetric tensor $A_{(4)}$ with self dual field strength from R-R sector, are eliminated by above projection.

Here we will consider the propagation of the open string in the background of type IIB theory. In order to clarify notation and terminology we will distinguish two descriptions of the same theory. We start with variables x^μ , θ^α and $\bar{\theta}^\alpha$ and background fields $G_{\mu\nu}$, $B_{\mu\nu}$, Ψ_μ^α , $\bar{\Psi}_\mu^\alpha$ and $F^{\alpha\beta}$ where the theory is described by equations of motion and boundary conditions. We are able to solve boundary conditions and introduce the effective theory defined only by equations of motion. It turns out that this effective theory is again the string theory, but in terms of effective coordinates and effective background fields. As a consequence of the boundary conditions, effective theory is 2π periodic and describes propagation of closed string in the background of type I theory.

In Ref.[1] we investigated the relation between type IIB and type I superstring theories in pure spinor formulation [3, 4]. It turned out that effective theory, obtained from initial one on the solution of boundary conditions, is just type I closed superstring theory. We improved known expressions for type I background fields with terms bilinear in Ω odd fields of type IIB superstring theory. In the present paper we extend the result of Ref.[1] embedding Dp -brane so that string endpoints move along it. Let us note that the action, which we used in Ref.[1] and in the present paper, can be obtained from general expression for type IIB superstring action [5] requiring that all background fields are constant and neglecting all nonquadratic terms. Consequently, all results we obtain are valid up to quadratic level.

We want to have stable Dp -branes [2] both in initial and final (effective) theory. Electrical charge and charge of magnetic dual brane in $D = 10$ are given by the following expressions, respectively

$$e_p = \int_{S^{8-p}} {}^*F_{(8-p)}, \quad \mu_{6-p} = \int_{S^{p+2}} F_{(p+2)}, \quad (1.1)$$

where $F_{(p+2)} = dA_{(p+1)}$ is field strength and ${}^*F_{(8-p)}$ is its Hodge dual. The R-R sector of type IIB theory contains gauge fields $A_{(0)}$, $A_{(2)}$ and $A_{(4)}$. In effective, type I superstring theory, only two form gauge field, $A_{(2)}$, exists. It couples electrically to $D1$ -brane (e_1) and magnetically to $D5$ -brane (μ_5). In order to work with stable Dp -branes in both theories, we will embed $D5$ -brane.

We choose Neumann boundary conditions for x^i coordinates ($i = 0, 1, \dots, 5$), and Dirichlet boundary conditions for the rest ones x^a ($a = 6, \dots, 9$). In this way we embed $D5$ -brane in type IIB and type I theories in $D = 10$, and break the initial symmetry $SO(1, 9)$ to $SO(1, 5) \times SO(4)$.

Any $D = 10$ dimensional Majorana-Weyl spinor S^α ($\alpha = 1, 2, \dots, 16$) can be expressed in terms of two $D5$ -brane opposite chirality Weyl spinors, S^{α_1} and S^{α_2} ($\alpha_1, \alpha_2 = 1, 2, \dots, 8$) [2, 6, 7]. According to this decomposition of spinors, the ten dimensional bispinor $F^{\alpha\beta}$ can be expressed in terms of 8 independent $D5$ -brane bispinors (Appendix B). It turns out that R-R sector on $D5$ -brane contains four scalars $a_{(0)}$, four vectors $a_{(1)}$ and four two rank antisymmetric tensors $a_{(2)}$ with self-dual field strengths. For fermionic $D5$ -brane coordinates we choose, $(\theta^{\alpha_1} - \bar{\theta}^{\alpha_1})|_0^\pi = 0$ and $(\theta^{\alpha_2} + \bar{\theta}^{\alpha_2})|_0^\pi = 0$, which produces corresponding boundary conditions for canonically conjugated momenta, $(\pi_{\alpha_1} - \bar{\pi}_{\alpha_1})|_0^\pi = 0$ and $(\pi_{\alpha_2} + \bar{\pi}_{\alpha_2})|_0^\pi = 0$.

In our approach boundary conditions are treated as canonical constraints. It turns out that all constraints originating from boundary conditions are of the second class. Solving the second class constraints, we obtain effective theory, which is described by the Ω even Lagrangian. We recognize effective theory as type I theory with embedded $D5$ -brane. Consequently, we obtain the expressions for $D5$ -brane background fields of type I superstring theory in terms of corresponding ones of type IIB. More precisely, $D5$ -brane background fields of type I theory, beside known term with Ω even fields, contain new term with quadratic combinations of Ω odd $D5$ -brane background fields of type IIB theory. The quadratic parts of effective background fields can be considered as supersymmetric generalization of the open string metric $G_{\mu\nu}^{eff}$ obtained by Seiberg and Witten [8].

The expressions for $D5$ -brane background fields of type I theory can be obtained directly from the expressions for effective background fields of Ref.[1] using T-dualities along x^a directions [9].

At the end we give some concluding remarks. Also there are four appendices. In Appendix A we introduce representation of gamma matrices adopted from first reference in [2] and then consider the spinors in ten and six dimensions and relations between them. The Appendix B deals with bispinors in ten and six dimensions. We showed that ten dimensional bispinor can be expressed in terms of eight independent $D5$ -brane bispinors. Discussion about complex coordinates and their canonically conjugated momenta is given in the Appendix C. The complete consistency procedure for fermionic constraints is presented in Appendix D.

2 Embedding $D5$ -brane in type IIB theory

In this section we will consider action for type IIB theory in pure spinor formulation [3, 5]. This theory is originally formulated using BRST charge and contains ghost fields. As in [4], we drop ghost terms and consider only ghost independent part of the action. We will preserve the parts of background fields which are nontrivial on the embedded $D5$ -brane, where the open string endpoints are attached.

Let us start with sigma model action for type IIB superstring of Ref.[5]

$$S = S_0 + V_{SG}. \quad (2.1)$$

The action in a flat background

$$S_0 = \int_{\Sigma} d^2\xi \left(\frac{\kappa}{2} \eta^{mn} \eta_{\mu\nu} \partial_m x^\mu \partial_n x^\nu - \pi_\alpha \partial_- \theta^\alpha + \partial_+ \bar{\theta}^\alpha \bar{\pi}_\alpha \right), \quad (2.2)$$

is deformed by integrated form of the massless IIB supergravity vertex operator

$$V_{SG} = \int_{\Sigma} d^2\xi X_M^T A_{MN} \bar{X}_N. \quad (2.3)$$

The world sheet Σ is parameterized by $\xi^m = (\xi^0 = \tau, \xi^1 = \sigma)$, and $D = 10$ dimensional space-time is parameterized by coordinates x^μ ($\mu = 0, 1, 2, \dots, 9$). The fermionic part of superspace is spanned by same chirality fermionic coordinates θ^α and $\bar{\theta}^\alpha$, while the variables π_α and $\bar{\pi}_\alpha$ are their canonically conjugated momenta. These fermionic coordinates and momenta are Majorana-Weyl spinors in $D = 10$ dimensions. We use notation: $\dot{X} \equiv \partial_\tau X$, $X' \equiv \partial_\sigma X$ and $\partial_\pm = \partial_\tau \pm \partial_\sigma$, where X is an arbitrary function of τ and σ .

For our purpose it is enough to consider the part of left and right supersymmetric tensors

$$X_M = \begin{pmatrix} \partial_+ \theta^\alpha \\ \Pi_+^\mu \\ d_\alpha \end{pmatrix}, \quad \bar{X}_N = \begin{pmatrix} \partial_- \bar{\theta}^\alpha \\ \Pi_-^\mu \\ \bar{d}_\alpha \end{pmatrix}, \quad (2.4)$$

and matrix

$$A_{MN} = \begin{pmatrix} A_{\alpha\beta} & A_{\alpha\nu} & E_\alpha^\beta \\ A_{\mu\beta} & A_{\mu\nu} & \bar{E}_\mu^\beta \\ E_\nu^\alpha & E_\nu^\beta & P^{\alpha\beta} \end{pmatrix}. \quad (2.5)$$

Matrix with superfields generally depends on x^μ , θ^α and $\bar{\theta}^\alpha$.

The BRST invariance of vertex operator produces equations of motion

$$\Gamma_{\mu\nu\rho\sigma\lambda}^{\alpha\beta} D_\alpha A_{\beta\gamma} = 0, \quad \Gamma_{\mu\nu\rho\sigma\lambda}^{\alpha\beta} \bar{D}_\alpha A_{\gamma\beta} = 0. \quad (2.6)$$

We will additionally require that all background fields in (2.5) are constant and restrict the analysis only to the quadratic terms. With these assumptions there exists simple solution

$$\Pi_\pm^\mu \rightarrow \partial_\pm x^\mu, \quad d_\alpha \rightarrow \pi_\alpha, \quad \bar{d}_\alpha \rightarrow \bar{\pi}_\alpha, \quad (2.7)$$

and only nontrivial superfields take the form

$$A_{\mu\nu} = \kappa \left(\frac{1}{2} g_{\mu\nu} + B_{\mu\nu} \right), \quad E_\nu^\alpha = -\Psi_\nu^\alpha, \quad \bar{E}_\mu^\alpha = \bar{\Psi}_\mu^\alpha, \quad P^{\alpha\beta} = \frac{1}{2\kappa} F^{\alpha\beta}, \quad (2.8)$$

where $g_{\mu\nu}$ is symmetric and $B_{\mu\nu}$ is antisymmetric tensor. We adopt expressions to be in agreement with our conventions.

Under imposed condition we obtain the vertex operator

$$V_{SG} = \int_{\Sigma} d^2\xi \left[\kappa \left(\frac{1}{2} g_{\mu\nu} + B_{\mu\nu} \right) \partial_+ x^\mu \partial_- x^\nu - \pi_\alpha \Psi_\mu^\alpha \partial_- x^\mu + \partial_+ x^\mu \bar{\Psi}_\mu^\alpha \bar{\pi}_\alpha + \frac{1}{2\kappa} \pi_\alpha F^{\alpha\beta} \pi_\beta \right]. \quad (2.9)$$

Together with flat background action it produces

$$S = \kappa \int_{\Sigma} d^2\xi \left[\frac{1}{2} \eta^{mn} G_{\mu\nu} + \varepsilon^{mn} B_{\mu\nu} \right] \partial_m x^\mu \partial_n x^\nu + \int_{\Sigma} d^2\xi \left[-\pi_\alpha \partial_- (\theta^\alpha + \Psi_\mu^\alpha x^\mu) + \partial_+ (\bar{\theta}^\alpha + \bar{\Psi}_\mu^\alpha x^\mu) \bar{\pi}_\alpha + \frac{1}{2\kappa} \pi_\alpha F^{\alpha\beta} \bar{\pi}_\beta \right], \quad (2.10)$$

where $G_{\mu\nu} = \eta_{\mu\nu} + g_{\mu\nu}$.

Choosing Neumann boundary conditions for x^i ($i = 0, 1, \dots, 5$) and Dirichlet boundary conditions for orthogonal directions x^a ($a = 6, 7, 8, 9$), we embed $D5$ -brane in $D = 10$ dimensional space-time. Orthogonality of these two sets of coordinates implies $G_{ia} = 0$. We assume that antisymmetric Neveu-Schwarz field $B_{\mu\nu}$ is nontrivial only along $D5$ -brane, $B_{\mu\nu} \rightarrow B_{ij}$. In NS-R sector the nonzero components Ψ_i^α and $\bar{\Psi}_i^\alpha$ can be expressed in terms of $D5$ -brane Weyl spinors $\Psi_i^{\alpha_1}$, $\Psi_i^{\alpha_2}$, $\bar{\Psi}_i^{\alpha_1}$ and $\bar{\Psi}_i^{\alpha_2}$ (see Appendix A). The rest ones Ψ_a^α and $\bar{\Psi}_a^\alpha$ are set to zero. In the R-R sector we assume nonzero value of all $D5$ -brane bispinors (see Appendix B). Because we restricted our analysis to quadratic terms, the part of the action describing the free string oscillation in x^a directions decouples from the rest. Taking into account all these assumptions, the action gets the form

$$\begin{aligned} S &= \kappa \int_{\Sigma} d^2\xi \left[\frac{1}{2} \eta^{mn} G_{ij} + \varepsilon^{mn} B_{ij} \right] \partial_m x^i \partial_n x^j \\ &+ 2\Re \left\{ \int_{\Sigma} d^2\xi \left[-\pi_{\alpha_1} (\partial_\tau - \partial_\sigma) (\theta^{\alpha_1} + \Psi_i^{\alpha_1} x^i) + (\partial_\tau + \partial_\sigma) (\bar{\theta}^{\alpha_1} + \bar{\Psi}_i^{\alpha_1} x^i) \bar{\pi}_{\alpha_1} \right] \right\} \\ &+ 2\Re \left\{ \int_{\Sigma} d^2\xi \left[-\pi_{\alpha_2} (\partial_\tau - \partial_\sigma) (\theta^{\alpha_2} + \Psi_i^{\alpha_2} x^i) + (\partial_\tau + \partial_\sigma) (\bar{\theta}^{\alpha_2} + \bar{\Psi}_i^{\alpha_2} x^i) \bar{\pi}_{\alpha_2} \right] \right\} \\ &+ \frac{1}{\kappa} \Re \left\{ \int_{\Sigma} d^2\xi \left[\pi_{\alpha_1} f_{11}^{\alpha_1\beta_1} \bar{\pi}_{\beta_1} + \pi_{\alpha_1} f_{14}^{\alpha_1\beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_2} f_{22}^{\alpha_2\beta_2} \bar{\pi}_{\beta_2} - \pi_{\alpha_2} f_{23}^{\alpha_2\beta_2} \bar{\pi}_{\beta_2}^* \right] \right\} \\ &+ \frac{1}{\kappa} \Re \left\{ \int_{\Sigma} d^2\xi \left[\pi_{\alpha_2} f_{21}^{\alpha_2\beta_1} \bar{\pi}_{\beta_1} - \pi_{\alpha_1} f_{12}^{\alpha_1\beta_2} \bar{\pi}_{\beta_2} + \pi_{\alpha_2} f_{24}^{\alpha_2\beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_1} f_{13}^{\alpha_1\beta_2} \bar{\pi}_{\beta_2}^* \right] \right\}, \end{aligned} \quad (2.11)$$

where \Re means real part of some complex number and $*$ means complex conjugation.

In Table 1 we summarize the list of the background fields of the type IIB superstring theory in $D = 10$ dimensional space-time, fields living on the $D5$ -brane and the rest fields.

Sector	x^μ ($\mu = 0, 1, \dots, 9$)	x^i ($i = 0, 1, \dots, 5$)	x^a ($a = 6, 7, 8, 9$)
NS-NS	$G_{\mu\nu}$ $B_{\mu\nu}$ $\Phi = 0$	G_{ij} B_{ij}	$G_{ia} = 0, G_{ab}(\text{decoupled})$ $B_{ia} = B_{ab} = 0$
NS-R	Ψ_μ^α $\bar{\Psi}_\mu^\alpha$	$\Psi_i^{\alpha_1}, \Psi_i^{\alpha_2}$ $\bar{\Psi}_i^{\alpha_1}, \bar{\Psi}_i^{\alpha_2}$	$\Psi_a^\alpha = 0$ $\bar{\Psi}_a^\alpha = 0$
R-R	$F^{\alpha\beta}$	$f_{11}^{\alpha_1\beta_1}, f_{22}^{\alpha_2\beta_2}, f_{14}^{\alpha_1\beta_1}, f_{23}^{\alpha_2\beta_2}$ $f_{12}^{\alpha_1\beta_2}, f_{21}^{\alpha_2\beta_1}, f_{13}^{\alpha_1\beta_2}, f_{24}^{\alpha_2\beta_1}$	

Table 1: Background fields of the type IIB theory: the complete set, part living on $D5$ -brane and the rest fields eliminated from the theory.

3 Canonical analysis of type IIB superstring theory with $D5$ -brane

In this section we will perform canonical analysis of pure spinor formulation of type IIB theory with embedded $D5$ -brane.

3.1 Canonical Hamiltonian

Momentum canonically conjugated to x^i has the form

$$\pi_i \equiv \frac{\partial \mathcal{L}}{\partial \dot{x}^i} = \kappa(G_{ij}\dot{x}^j - 2B_{ij}x'^j) + 2\Re(-\pi_{\alpha_1}\Psi_i^{\alpha_1} - \pi_{\alpha_2}\Psi_i^{\alpha_2} + \bar{\Psi}_i^{\alpha_1}\bar{\pi}_{\alpha_1} + \bar{\Psi}_i^{\alpha_2}\bar{\pi}_{\alpha_2}), \quad (3.1)$$

while by definition the momenta π_{α_1} , π_{α_2} , $\bar{\pi}_{\alpha_1}$ and $\bar{\pi}_{\alpha_2}$ are canonically conjugated to the coordinates θ^{α_1} , θ^{α_2} , $\bar{\theta}^{\alpha_1}$ and $\bar{\theta}^{\alpha_2}$, respectively. In Appendix C we discussed this connection between complex coordinates and momenta.

According to the definition of canonical Hamiltonian

$$\mathcal{H}_c = \dot{x}^i \pi_i + 2\Re(\dot{\theta}^{\alpha_1}\pi_{\alpha_1} + \dot{\bar{\theta}}^{\alpha_1}\bar{\pi}_{\alpha_1} + \dot{\theta}^{\alpha_2}\pi_{\alpha_2} + \dot{\bar{\theta}}^{\alpha_2}\bar{\pi}_{\alpha_2}) - \mathcal{L}, \quad (3.2)$$

we have

$$H_c = \int d\sigma \mathcal{H}_c, \quad \mathcal{H}_c = T_- - T_+, \quad T_\pm = t_\pm - \tau_\pm, \quad (3.3)$$

where

$$\begin{aligned} t_\pm &= \mp \frac{1}{4\kappa} G^{ij} I_{\pm i} I_{\pm j}, \\ I_{\pm i} &= \pi_i + 2\kappa \Pi_{\pm ij} x'^j + 2\Re(\pi_{\alpha_1} \Psi_i^{\alpha_1} + \pi_{\alpha_2} \Psi_i^{\alpha_2} - \bar{\Psi}_i^{\alpha_1} \bar{\pi}_{\alpha_1} - \bar{\Psi}_i^{\alpha_2} \bar{\pi}_{\alpha_2}), \\ \tau_+ &= 2\Re[(\theta'^{\alpha_1} + \Psi_i^{\alpha_1} x'^i) \pi_{\alpha_1} + (\theta'^{\alpha_2} + \Psi_i^{\alpha_2} x'^i) \pi_{\alpha_2}] \\ &\quad - \frac{1}{2\kappa} \Re(\pi_{\alpha_1} f_{11}^{\alpha_1\beta_1} \bar{\pi}_{\beta_1} + \pi_{\alpha_1} f_{14}^{\alpha_1\beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_2} f_{22}^{\alpha_2\beta_2} \bar{\pi}_{\beta_2} - \pi_{\alpha_2} f_{23}^{\alpha_2\beta_2} \bar{\pi}_{\beta_2}^*) \end{aligned}$$

$$\begin{aligned}
& - \frac{1}{2\kappa} \Re \left(\pi_{\alpha_2} f_{21}^{\alpha_2 \beta_1} \bar{\pi}_{\beta_1} - \pi_{\alpha_1} f_{12}^{\alpha_1 \beta_2} \bar{\pi}_{\beta_2} + \pi_{\alpha_2} f_{24}^{\alpha_2 \beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_1} f_{13}^{\alpha_1 \beta_2} \bar{\pi}_{\beta_2}^* \right), \\
\tau_- &= 2\Re \left[(\bar{\theta}'^{\alpha_1} + \bar{\Psi}_i^{\alpha_1} x'^i) \bar{\pi}_{\alpha_1} + (\bar{\theta}'^{\alpha_2} + \bar{\Psi}_i^{\alpha_2} x'^i) \bar{\pi}_{\alpha_2} \right] \\
&+ \frac{1}{2\kappa} \Re \left(\pi_{\alpha_1} f_{11}^{\alpha_1 \beta_1} \bar{\pi}_{\beta_1} + \pi_{\alpha_1} f_{14}^{\alpha_1 \beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_2} f_{22}^{\alpha_2 \beta_2} \bar{\pi}_{\beta_2} - \pi_{\alpha_2} f_{23}^{\alpha_2 \beta_2} \bar{\pi}_{\beta_2}^* \right) \\
&+ \frac{1}{2\kappa} \Re \left(\pi_{\alpha_2} f_{21}^{\alpha_2 \beta_1} \bar{\pi}_{\beta_1} - \pi_{\alpha_1} f_{12}^{\alpha_1 \beta_2} \bar{\pi}_{\beta_2} + \pi_{\alpha_2} f_{24}^{\alpha_2 \beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_1} f_{13}^{\alpha_1 \beta_2} \bar{\pi}_{\beta_2}^* \right). \quad (3.4)
\end{aligned}$$

Using the standard Poisson bracket algebra

$$\{x^i(\sigma), \pi_j(\bar{\sigma})\} = \delta^i_j \delta(\sigma - \bar{\sigma}), \quad (3.5)$$

$$\begin{aligned}
\{\theta^{\alpha_1}(\sigma), \pi_{\beta_1}(\bar{\sigma})\} &= -\delta^{\alpha_1}_{\beta_1} \delta(\sigma - \bar{\sigma}), \quad \{\bar{\theta}^{\alpha_1}(\sigma), \bar{\pi}_{\beta_1}(\bar{\sigma})\} = -\delta^{\alpha_1}_{\beta_1} \delta(\sigma - \bar{\sigma}), \\
\{\theta^{\alpha_2}(\sigma), \pi_{\beta_2}(\bar{\sigma})\} &= -\delta^{\alpha_2}_{\beta_2} \delta(\sigma - \bar{\sigma}), \quad \{\bar{\theta}^{\alpha_2}(\sigma), \bar{\pi}_{\beta_2}(\bar{\sigma})\} = -\delta^{\alpha_2}_{\beta_2} \delta(\sigma - \bar{\sigma}), \quad (3.6)
\end{aligned}$$

we calculate the algebra of currents

$$\{I_{\pm i}(\sigma), I_{\pm j}(\bar{\sigma})\} = \pm 2\kappa G_{ij} \delta', \quad \{I_{\pm i}(\sigma), I_{\mp j}(\bar{\sigma})\} = 0. \quad [\delta' \equiv \partial_\sigma \delta(\sigma - \bar{\sigma})] \quad (3.7)$$

With the help of this algebra we find that components T_\pm satisfy Virasoro algebra

$$\{T_\pm(\sigma), T_\pm(\bar{\sigma})\} = -[T_\pm(\sigma) + T_\pm(\bar{\sigma})] \delta', \quad \{T_\pm(\sigma), T_\mp(\bar{\sigma})\} = 0. \quad (3.8)$$

The Poisson bracket between canonical Hamiltonian and current $I_{\pm i}$ is proportional to its sigma derivative

$$\{H_c, I_{\pm i}\} = \mp I'_{\pm i}. \quad (3.9)$$

3.2 Boundary conditions as canonical constraints

Following method of Ref.[10], using canonical approach, we will derive boundary conditions directly in terms of canonical variables. Varying Hamiltonian H_c we obtain

$$\delta H_c = \delta H_c^{(R)} - \left[\gamma_i^{(0)} \delta x^i + 2\Re(\pi_{\alpha_1} \delta \theta^{\alpha_1} + \delta \bar{\theta}^{\alpha_1} \bar{\pi}_{\alpha_1}) + 2\Re(\pi_{\alpha_2} \delta \theta^{\alpha_2} + \delta \bar{\theta}^{\alpha_2} \bar{\pi}_{\alpha_2}) \right] \Big|_0^\pi, \quad (3.10)$$

where $\delta H_c^{(R)}$ is regular term of the form

$$\begin{aligned}
\delta H_c^{(R)} &= \int d\sigma [A_i \delta x^i + B^i \delta \pi_i + 2\Re(C_{\alpha_1} \delta \theta^{\alpha_1} + D_{\alpha_1} \delta \bar{\theta}^{\alpha_1} + E^{\alpha_1} \delta \pi_{\alpha_1} + F^{\alpha_1} \delta \bar{\pi}_{\alpha_1})] \\
&+ 2\Re \int d\sigma (C_{\alpha_2} \delta \theta^{\alpha_2} + D_{\alpha_2} \delta \bar{\theta}^{\alpha_2} + E^{\alpha_2} \delta \pi_{\alpha_2} + F^{\alpha_2} \delta \bar{\pi}_{\alpha_2}), \quad (3.11)
\end{aligned}$$

and

$$\gamma_i^{(0)} = \Pi_{+i}^j I_{-j} + \Pi_{-i}^j I_{+j} + 2\Re(\pi_{\alpha_1} \Psi_i^{\alpha_1} + \pi_{\alpha_2} \Psi_i^{\alpha_2} + \bar{\Psi}_i^{\alpha_1} \bar{\pi}_{\alpha_1} + \bar{\Psi}_i^{\alpha_2} \bar{\pi}_{\alpha_2}). \quad (3.12)$$

As a time translation generator Hamiltonian must have well defined derivatives with respect to its variables. Consequently, boundary term has to vanish and we obtain

$$\left[\gamma_i^{(0)} \delta x^i + 2\Re(\pi_{\alpha_1} \delta \theta^{\alpha_1} + \delta \bar{\theta}^{\alpha_1} \bar{\pi}_{\alpha_1}) + 2\Re(\pi_{\alpha_2} \delta \theta^{\alpha_2} + \delta \bar{\theta}^{\alpha_2} \bar{\pi}_{\alpha_2}) \right] \Big|_0^\pi = 0. \quad (3.13)$$

For bosonic coordinates x^i we choose Neumann boundary conditions, implying

$$\gamma_i^{(0)} \Big|_0^\pi = 0, \quad (3.14)$$

while for fermionic coordinates we choose

$$(\theta^{\alpha_1} - \bar{\theta}^{\alpha_1}) \Big|_0^\pi = 0, \quad (\theta^{\alpha_2} + \bar{\theta}^{\alpha_2}) \Big|_0^\pi = 0, \quad (3.15)$$

which produces additional boundary conditions

$$(\pi_{\alpha_1} - \bar{\pi}_{\alpha_1}) \Big|_0^\pi = 0, \quad (\pi_{\alpha_2} + \bar{\pi}_{\alpha_2}) \Big|_0^\pi = 0. \quad (3.16)$$

According with Refs.[10, 11, 12], we will treat the expressions (3.14)-(3.16) as canonical constraints.

3.3 Consistency of bosonic constraints

Using Eq.(3.9) and standard Poisson algebra, the consistency procedure for $\gamma_i^{(0)}$ produces an infinite set of constraints $\gamma_i^{(n)}$ ($n = 1, 2, \dots$) with

$$\begin{aligned} \gamma_i^{(n)} &\equiv \{H_c, \gamma_i^{(n-1)}\} = \Pi_{+i}^j \partial_\sigma^{(n)} I_{-j} + (-1)^n \Pi_{-i}^j \partial_\sigma^{(n)} I_{+j} \\ &+ 2\Re \left[(-1)^n \partial_\sigma^{(n)} \pi_{\alpha_1} \Psi_i^{\alpha_1} + (-1)^n \partial_\sigma^{(n)} \pi_{\alpha_2} \Psi_i^{\alpha_2} + \bar{\Psi}_i^{\alpha_1} \partial_\sigma^{(n)} \bar{\pi}_{\alpha_1} + \bar{\Psi}_i^{\alpha_2} \partial_\sigma^{(n)} \bar{\pi}_{\alpha_2} \right]. \end{aligned} \quad (3.17)$$

They can be rewritten in the compact σ -dependent form

$$\begin{aligned} \Gamma_i(\sigma) &\equiv \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} \gamma_i^{(n)}(\sigma=0) = \Pi_{+i}^j I_{-j}(\sigma) + \Pi_{-i}^j I_{+j}(-\sigma) \\ &+ 2\Re \left[\pi_{\alpha_1}(-\sigma) \Psi_i^{\alpha_1} + \pi_{\alpha_2}(-\sigma) \Psi_i^{\alpha_2} + \bar{\Psi}_i^{\alpha_1} \bar{\pi}_{\alpha_1}(\sigma) + \bar{\Psi}_i^{\alpha_2} \bar{\pi}_{\alpha_2}(\sigma) \right] \\ &= \tilde{p}_i + 2(BG^{-1})_i^j p_j - \kappa G_{ij}^{eff} \tilde{q}^j + 4\Re \left[\Pi_{+i}^j P_s \pi_{\alpha_1}(\sigma) \Psi_j^{\alpha_1} + \Pi_{+i}^j P_s \pi_{\alpha_2}(\sigma) \Psi_j^{\alpha_2} \right] \\ &- 4\Re \left[\Pi_{-i}^j \bar{\Psi}_j^{\alpha_1} P_s \bar{\pi}_{\alpha_1}(\sigma) + \Pi_{-i}^j \bar{\Psi}_j^{\alpha_2} P_s \bar{\pi}_{\alpha_2}(\sigma) \right]. \end{aligned} \quad (3.18)$$

Here we introduced new variables, even and odd under world-sheet parity transformation $\Omega : \sigma \rightarrow -\sigma$. For bosonic variables we use standard notation [11, 12]

$$q^i(\sigma) = P_s x^i(\sigma) \equiv \frac{1}{2} [x^i(\sigma) + x^i(-\sigma)], \quad \tilde{q}^i(\sigma) = P_a x^i(\sigma) \equiv \frac{1}{2} [x^i(\sigma) - x^i(-\sigma)], \quad (3.19)$$

$$p_i(\sigma) = P_s \pi_i(\sigma) \equiv \frac{1}{2} [\pi_i(\sigma) + \pi_i(-\sigma)], \quad \tilde{p}_i(\sigma) = P_a \pi_i(\sigma) \equiv \frac{1}{2} [\pi_i(\sigma) - \pi_i(-\sigma)], \quad (3.20)$$

while for fermionic ones we explicitly use the projectors on Ω even and odd parts

$$P_s = \frac{1}{2}(1 + \Omega), \quad P_a = \frac{1}{2}(1 - \Omega). \quad (3.21)$$

3.4 Consistency of fermionic constraints

The complete consistency procedure is given in Appendix D. Here we write only the form of the fermionic constraints after Dirac consistency procedure.

We start consistency procedure for fermionic constraints, (3.15) and (3.16), applying this procedure to the variables

$$A^{(0)} = (\theta^{\alpha_1}, \theta^{\alpha_2}, \bar{\theta}^{\alpha_1}, \bar{\theta}^{\alpha_2}, \pi_{\alpha_1}, \pi_{\alpha_2}, \bar{\pi}_{\alpha_1}, \bar{\pi}_{\alpha_2}),$$

and obtain an infinite set of the constraints $A^{(n)}$ ($n = 0, 1, 2, 3, \dots$). The complete set of constraints following from boundary conditions (3.15) and Dirac canonical procedure, in compact notation at $\sigma = 0$ has the form

$$\Gamma^{\alpha_1}(\sigma) = \Theta^{\alpha_1}(\sigma) - \bar{\Theta}^{\alpha_1}(\sigma), \quad \Gamma^{\alpha_2}(\sigma) = \Theta^{\alpha_2}(\sigma) + \bar{\Theta}^{\alpha_2}(\sigma), \quad (3.22)$$

where right-hand side variables are defined in (D.8)-(D.9). Similarly, from (3.16), we have

$$\begin{aligned} \Gamma_{\alpha_1}^{\pi}(\sigma) &\equiv \Pi_{\alpha_1}(\sigma) - \bar{\Pi}_{\alpha_1}(\sigma) = \pi_{\alpha_1}(-\sigma) - \bar{\pi}_{\alpha_1}(\sigma), \\ \Gamma_{\alpha_2}^{\pi}(\sigma) &\equiv \Pi_{\alpha_2}(\sigma) + \bar{\Pi}_{\alpha_2}(\sigma) = \pi_{\alpha_2}(-\sigma) + \bar{\pi}_{\alpha_2}(\sigma), \end{aligned} \quad (3.23)$$

where Π_{α_1} , Π_{α_2} , $\bar{\Pi}_{\alpha_1}$ and $\bar{\Pi}_{\alpha_2}$ are defined in Eq.(D.10).

For all bosonic and fermionic constraints we also apply the consistency procedure at $\sigma = \pi$ and obtain similar expressions, where all variables depending on $-\sigma$ are replaced by the same variables depending on $2\pi - \sigma$. That set of constraints is solved by 2π periodicity of all canonical variables as well as in Refs.[11, 12].

3.5 Classification of constraints

Let us denote all constraints with $\Gamma_A = (\Gamma_i, \Gamma^{\alpha_1}, \Gamma^{\alpha_2}, \Gamma_{\alpha_1}^{\pi}, \Gamma_{\alpha_2}^{\pi})$. From

$$\{H_c, \Gamma_A\} = \Gamma'_A \approx 0, \quad (3.24)$$

it follows that all constraints weakly commute with canonical Hamiltonian, so there are no more constraints in the theory and the consistency procedure is completed.

We are going to classify the constraints. For practical reasons, in order to classify the constraints easier, let us first consider the quantities Γ'^{α_1} and Γ'^{α_2} instead Γ^{α_1} and Γ^{α_2} , as constraints in the theory. The algebra of the constraints ${}^*\Gamma_A = (\Gamma_i, \Gamma'^{\alpha_1}, \Gamma'^{\alpha_2}, \Gamma_{\alpha_1}^{\pi}, \Gamma_{\alpha_2}^{\pi})$ has the form

$$\{{}^*\Gamma_A, {}^*\Gamma_B\} = M_{AB}\delta', \quad (3.25)$$

where the supermatrix M_{AB} is given by the expression

$$\begin{aligned}
M_{AB} &= \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix} = \left(\begin{array}{c|cccc} (M_1)_{ij} & M_i^{\gamma_1} & M_i^{\gamma_2} & \bar{M}_{i\delta_1} & \bar{M}_{i\delta_2} \\ \hline M^{\alpha_1}_j & M^{\alpha_1\gamma_1} & M^{\alpha_1\gamma_2} & \bar{M}^{\alpha_1}_{\delta_1} & \bar{M}^{\alpha_1}_{\delta_2} \\ M^{\alpha_2}_j & M^{\alpha_2\gamma_1} & M^{\alpha_2\gamma_2} & \bar{M}^{\alpha_2}_{\delta_1} & \bar{M}^{\alpha_2}_{\delta_2} \\ \hline \bar{M}_{\beta_1 j} & \bar{M}_{\beta_1}^{\gamma_1} & \bar{M}_{\beta_1}^{\gamma_2} & \tilde{M}_{\beta_1\delta_1} & \tilde{M}_{\beta_1\delta_2} \\ \bar{M}_{\beta_2 j} & \bar{M}_{\beta_2}^{\gamma_1} & \bar{M}_{\beta_2}^{\gamma_2} & \tilde{M}_{\beta_2\delta_1} & \tilde{M}_{\beta_2\delta_2} \end{array} \right) \\
&= \left(\begin{array}{c|ccccc} -\kappa G_{ij}^{eff} & -2(\Psi^{eff})_i^{\gamma_1} & -2(\Psi^{eff})_i^{\gamma_2} & 0 & 0 \\ \hline -2(\Psi^{eff})^{\alpha_1}_j & \frac{1}{\kappa}(f_{11}^{eff})^{\alpha_1\gamma_1} & \frac{1}{\kappa}(f_{12}^{eff})^{\alpha_1\gamma_2} & -2\delta^{\alpha_1}_{\delta_1} & 0 \\ -2(\Psi^{eff})^{\alpha_2}_j & -\frac{1}{\kappa}(f_{12}^{eff})^{\alpha_2\gamma_1} & \frac{1}{\kappa}(f_{22}^{eff})^{\alpha_2\gamma_2} & 0 & -2\delta^{\alpha_2}_{\delta_2} \\ \hline 0 & -2\delta_{\beta_1}^{\gamma_1} & 0 & 0 & 0 \\ 0 & 0 & -2\delta_{\beta_2}^{\gamma_2} & 0 & 0 \end{array} \right). \tag{3.26}
\end{aligned}$$

To the fields appearing in matrix M_{AB} we will refer as the effective background fields. They are defined as

$$\begin{aligned}
G_{ij}^{eff} &= G_{ij} - 4B_{ik}G^{kl}B_{lj}, \\
(\Psi_{eff})_i^{\alpha_1} &= \frac{1}{2}\Psi_{+i}^{\alpha_1} + B_{ik}G^{kj}\Psi_{-j}^{\alpha_1}, \quad (\Psi_{eff})_i^{\alpha_2} = \frac{1}{2}\Psi_{-i}^{\alpha_2} + B_{ik}G^{kj}\Psi_{+j}^{\alpha_2}, \\
(f_{11}^{eff})^{\alpha_1\beta_1} &= (f_{11}^a)^{\alpha_1\beta_1} - \Psi_{-i}^{\alpha_1}G^{ij}\Psi_{-j}^{\beta_1}, \quad (f_{22}^{eff})^{\alpha_2\beta_2} = (f_{22}^a)^{\alpha_2\beta_2} - \Psi_{+i}^{\alpha_2}G^{ij}\Psi_{+j}^{\beta_2}, \\
(f_{12}^{eff})^{\alpha_1\beta_2} &= \frac{1}{2}\left(f_{12}^{\alpha_1\beta_2} - f_{21}^{\beta_2\alpha_1}\right) - \Psi_{-i}^{\alpha_1}G^{ij}\Psi_{+j}^{\beta_2}, \tag{3.27}
\end{aligned}$$

where

$$\begin{aligned}
\Psi_{\pm i}^{\alpha_1} &= \Psi_i^{\alpha_1} \pm \bar{\Psi}_i^{\alpha_1}, \\
(f^s)^{\alpha_1\beta_1} &= \frac{1}{2}(f^{\alpha_1\beta_1} + f^{\beta_1\alpha_1}), \quad (f^a)^{\alpha_1\beta_1} = \frac{1}{2}(f^{\alpha_1\beta_1} - f^{\beta_1\alpha_1}). \tag{3.28}
\end{aligned}$$

The matrix M_{AB} is independent of the field strengths $f_{14}^{\alpha_1\beta_2}$, $f_{23}^{\alpha_2\beta_2}$, $f_{13}^{\alpha_1\beta_2}$ and $f_{24}^{\alpha_2\beta_1}$ because boundary conditions depend on complex conjugated momenta π^* but not on the corresponding coordinates θ^* .

From the definition of superdeterminant

$$s \det M_{AB} = \frac{\det(M_1 - M_2 M_4^{-1} M_3)}{\det M_4}, \tag{3.29}$$

and using the fact that

$$(M_2 M_4^{-1} M_3)_{ij} = 0, \quad \det M_4 = const., \tag{3.30}$$

we obtain

$$s \det M_{AB} \sim \det G_{ij}^{eff}. \tag{3.31}$$

Because we assume that effective metric G_{ij}^{eff} is nonsingular, we conclude that constraints ${}^*\Gamma_A$ are of the second class.

The original constraints Γ^{α_1} and Γ^{α_2} have the form (see Appendix D)

$$\begin{aligned}\Gamma^{\alpha_1} &= (\theta^{\alpha_1} - \bar{\theta}^{\alpha_1})|_{\sigma=0} + \sum_{n=1}^{\infty} \frac{\sigma^n}{n!} (\Theta_{(n)}^{\alpha_1} - \bar{\Theta}_{(n)}^{\alpha_1})(\sigma=0), \\ \Gamma^{\alpha_2} &= (\theta^{\alpha_2} + \bar{\theta}^{\alpha_2})|_{\sigma=0} + \sum_{n=1}^{\infty} \frac{\sigma^n}{n!} (\Theta_{(n)}^{\alpha_2} + \bar{\Theta}_{(n)}^{\alpha_2})(\sigma=0).\end{aligned}$$

The second class constraints Γ'^{α_1} and Γ'^{α_2} are effectively the derivatives of the parts under the sums. It is easy to check that zero modes, $(\theta^{\alpha_1} - \bar{\theta}^{\alpha_1})|_{\sigma=0}$ and $(\theta^{\alpha_2} + \bar{\theta}^{\alpha_2})|_{\sigma=0}$, are also second class constraints, and consequently, the complete set of the constraints, Γ_A , is of the second class.

The reason for having two steps in this consideration is that Poisson brackets of constraints ${}^*\Gamma_A$ closed on δ' function while those with zero modes closed on δ function.

4 Solution of the constraints

The solution of the second class constraints originating from boundary conditions, $\Gamma_i = 0$, $\Gamma^{\alpha_1} = 0$, $\Gamma^{\alpha_2} = 0$, $\Gamma_{\alpha_1}^{\pi} = 0$ and $\Gamma_{\alpha_2}^{\pi} = 0$, has the form

$$x^i(\sigma) = q^i - 2\Theta^{ij} \int_0^{\sigma} d\sigma_1 p_j + 4\Re \left(\Theta^{i\alpha_1} \int_0^{\sigma} d\sigma_1 p_{\alpha_1} + \Theta^{i\alpha_2} \int_0^{\sigma} d\sigma_1 p_{\alpha_2} \right), \quad \pi_i = p_i, \quad (4.1)$$

$$\begin{aligned}\theta^{\alpha_1}(\sigma) &= \frac{1}{2}(\xi^{\alpha_1} + \tilde{\xi}^{\alpha_1}) - \Theta^{i\alpha_1} \int_0^{\sigma} d\sigma_1 p_i - \Theta^{\alpha_1\beta_1} \int_0^{\sigma} d\sigma_1 p_{\beta_1} - \Theta^{\alpha_1\beta_2} \int_0^{\sigma} d\sigma_1 p_{\beta_2} \\ &\quad - {}^*\Theta^{\alpha_1\beta_1} \int_0^{\sigma} d\sigma_1 p_{\beta_1}^* - {}^*\Theta^{\alpha_1\beta_2} \int_0^{\sigma} d\sigma_1 p_{\beta_2}^*, \\ \pi_{\alpha_1} &= p_{\alpha_1} + \tilde{p}_{\alpha_1},\end{aligned} \quad (4.2)$$

$$\begin{aligned}\theta^{\alpha_2}(\sigma) &= \frac{1}{2}(\xi^{\alpha_2} + \tilde{\xi}^{\alpha_2}) - \Theta^{i\alpha_2} \int_0^{\sigma} d\sigma_1 p_i - \Theta^{\alpha_2\beta_1} \int_0^{\sigma} d\sigma_1 p_{\beta_1} - \Theta^{\alpha_2\beta_2} \int_0^{\sigma} d\sigma_1 p_{\beta_2} \\ &\quad - {}^*\Theta^{\alpha_2\beta_1} \int_0^{\sigma} d\sigma_1 p_{\beta_1}^* - {}^*\Theta^{\alpha_2\beta_2} \int_0^{\sigma} d\sigma_1 p_{\beta_2}^*, \\ \pi_{\alpha_2} &= p_{\alpha_2} + \tilde{p}_{\alpha_2},\end{aligned} \quad (4.3)$$

$$\begin{aligned}\bar{\theta}^{\alpha_1}(\sigma) &= \theta^{\alpha_1}(\sigma) - \tilde{\xi}^{\alpha_1}(\sigma) = \frac{1}{2}(\xi^{\alpha_1} - \tilde{\xi}^{\alpha_1}) - \Theta^{i\alpha_1} \int_0^{\sigma} d\sigma_1 p_i \\ &\quad - \Theta^{\alpha_1\beta_1} \int_0^{\sigma} d\sigma_1 p_{\beta_1} - \Theta^{\alpha_1\beta_2} \int_0^{\sigma} d\sigma_1 p_{\beta_2} - {}^*\Theta^{\alpha_1\beta_1} \int_0^{\sigma} d\sigma_1 p_{\beta_1}^* - {}^*\Theta^{\alpha_1\beta_2} \int_0^{\sigma} d\sigma_1 p_{\beta_2}^*, \\ \bar{\pi}_{\alpha_1} &= p_{\alpha_1} - \tilde{p}_{\alpha_1},\end{aligned} \quad (4.4)$$

$$\begin{aligned}
\bar{\theta}^{\alpha_2}(\sigma) &= -\theta^{\alpha_2}(\sigma) + \tilde{\xi}^{\alpha_2}(\sigma) = -\frac{1}{2}(\xi^{\alpha_2} - \tilde{\xi}^{\alpha_2}) + \Theta^{i\alpha_2} \int_0^\sigma d\sigma_1 p_i \\
&+ \Theta^{\alpha_2\beta_1} \int_0^\sigma d\sigma_1 p_{\beta_1} + \Theta^{\alpha_2\beta_2} \int_0^\sigma d\sigma_1 p_{\beta_2} + {}^*\Theta^{\alpha_2\beta_1} \int_0^\sigma d\sigma_1 p_{\beta_1}^* + {}^*\Theta^{\alpha_2\beta_2} \int_0^\sigma d\sigma_1 p_{\beta_2}^*, \\
\bar{\pi}_{\alpha_2} &= -p_{\alpha_2} + \tilde{p}_{\alpha_2},
\end{aligned} \tag{4.5}$$

in terms of the bosonic Ω even and odd variables given in (3.20) and the fermionic variables

$$\begin{aligned}
\frac{1}{2}\xi^{\alpha_1} &\equiv P_s \theta^{\alpha_1} = P_s \bar{\theta}^{\alpha_1}, \quad \tilde{\xi}^{\alpha_1} \equiv P_a(\theta^{\alpha_1} - \bar{\theta}^{\alpha_1}), \\
\frac{1}{2}\xi^{\alpha_2} &\equiv P_s \theta^{\alpha_2} = -P_s \bar{\theta}^{\alpha_2}, \quad \tilde{\xi}^{\alpha_2} \equiv P_a(\theta^{\alpha_2} + \bar{\theta}^{\alpha_2}), \\
p_{\alpha_1} &\equiv P_s \pi_{\alpha_1} = P_s \bar{\pi}_{\alpha_1}, \quad \tilde{p}_{\alpha_1} \equiv P_a \pi_{\alpha_1} = -P_a \bar{\pi}_{\alpha_1}, \\
p_{\alpha_2} &\equiv P_s \pi_{\alpha_2} = -P_s \bar{\pi}_{\alpha_2}, \quad \tilde{p}_{\alpha_2} \equiv P_a \pi_{\alpha_2} = P_a \bar{\pi}_{\alpha_2}.
\end{aligned} \tag{4.6}$$

The coefficients multiplying momenta are defined as

$$\Theta^{ij} = -\frac{1}{\kappa}(G_{eff}^{-1}BG^{-1})^{ij}, \tag{4.7}$$

$$\Theta^{i\alpha_1} = 2\Theta^{ij}(\Psi_{eff})_j^{\alpha_1} - \frac{1}{2\kappa}G^{ij}\Psi_{-j}^{\alpha_1}, \quad \Theta^{i\alpha_2} = 2\Theta^{ij}(\Psi_{eff})_j^{\alpha_2} - \frac{1}{2\kappa}G^{ij}\Psi_{+j}^{\alpha_2}, \tag{4.8}$$

$$\begin{aligned}
\Theta^{\alpha_1\beta_1} &= \frac{1}{2\kappa}(f_{11}^s)^{\alpha_1\beta_1} + 4(\Psi_{eff})_i^{\alpha_1}\Theta^{ij}(\Psi_{eff})_j^{\beta_1} - \frac{1}{\kappa}\Psi_{-i}^{\alpha_1}(G^{-1}BG^{-1})^{ij}\Psi_{-j}^{\beta_1} \\
&+ \frac{G^{ij}}{\kappa} \left[\Psi_{-i}^{\alpha_1}(\Psi_{eff})_j^{\beta_1} + \Psi_{-i}^{\beta_1}(\Psi_{eff})_j^{\alpha_1} \right],
\end{aligned} \tag{4.9}$$

$$\begin{aligned}
\Theta^{\alpha_1\beta_2} &= \Theta^{\beta_2\alpha_1} = \frac{1}{4\kappa}(f_{12}^{\alpha_1\beta_2} + f_{21}^{\beta_2\alpha_1}) - 4(\Psi_{eff})_i^{\alpha_1}\Theta^{ij}(\Psi_{eff})_j^{\beta_2} \\
&- \frac{1}{\kappa}\Psi_{-i}^{\alpha_1}(G^{-1}BG^{-1})^{ij}\Psi_{+j}^{\beta_2} + \frac{G^{ij}}{\kappa} \left[\Psi_{-i}^{\alpha_1}(\Psi_{eff})_j^{\beta_2} + \Psi_{+i}^{\beta_2}(\Psi_{eff})_j^{\alpha_1} \right],
\end{aligned} \tag{4.10}$$

$$\begin{aligned}
\Theta^{\alpha_2\beta_2} &= \frac{1}{2\kappa}(f_{22}^s)^{\alpha_2\beta_2} + 4(\Psi_{eff})_i^{\alpha_2}\Theta^{ij}(\Psi_{eff})_j^{\beta_2} - \frac{1}{\kappa}\Psi_{+i}^{\alpha_2}(G^{-1}BG^{-1})^{ij}\Psi_{+j}^{\beta_2} \\
&+ \frac{G^{ij}}{\kappa} \left[\Psi_{+i}^{\alpha_2}(\Psi_{eff})_j^{\beta_2} + \Psi_{+i}^{\beta_2}(\Psi_{eff})_j^{\alpha_2} \right],
\end{aligned} \tag{4.11}$$

$$\begin{aligned}
{}^*\Theta^{\alpha_1\beta_1} &= \frac{1}{4\kappa}(f_{14}^{\alpha_1\beta_1} + f_{14}^{*\beta_1\alpha_1}) + 4(\Psi_{eff})_i^{\alpha_1}\Theta^{ij}(\Psi_{eff})_j^{*\beta_1} - \frac{1}{\kappa}\Psi_{-i}^{\alpha_1}(G^{-1}BG^{-1})^{ij}\Psi_{-j}^{*\beta_1} \\
&+ \frac{G^{ij}}{\kappa} \left[\Psi_{-i}^{\alpha_1}(\Psi_{eff})_j^{*\beta_1} + \Psi_{-i}^{*\beta_1}(\Psi_{eff})_j^{\alpha_1} \right],
\end{aligned} \tag{4.12}$$

$$\begin{aligned}
{}^*\Theta^{\alpha_1\beta_2} &= {}^*\Theta^{\beta_2\alpha_1} = \frac{1}{4\kappa}(f_{13}^{\alpha_1\beta_2} + f_{24}^{*\beta_2\alpha_1}) - 4(\Psi_{eff})_i^{\alpha_1}\Theta^{ij}(\Psi_{eff})_j^{*\beta_2} \\
&- \frac{1}{\kappa}\Psi_{-i}^{\alpha_1}(G^{-1}BG^{-1})^{ij}\Psi_{+j}^{*\beta_2} + \frac{G^{ij}}{\kappa} \left[\Psi_{-i}^{\alpha_1}(\Psi_{eff})_j^{*\beta_2} + \Psi_{+i}^{*\beta_2}(\Psi_{eff})_j^{\alpha_1} \right],
\end{aligned} \tag{4.13}$$

$$\begin{aligned}
\Theta^{\alpha_2\beta_2} &= \frac{1}{4\kappa}(f_{23}^{\alpha_2\beta_2} + f_{23}^{\beta_2\alpha_2}) + 4(\Psi_{eff})_i^{\alpha_2}\Theta^{ij}(\Psi_{eff})_j^{*\beta_2} - \frac{1}{\kappa}\Psi_{+i}^{\alpha_2}(G^{-1}BG^{-1})^{ij}\Psi_{+j}^{*\beta_2} \\
&+ \frac{G^{ij}}{\kappa}\left[\Psi_{+i}^{\alpha_2}(\Psi_{eff})_j^{*\beta_2} + \Psi_{+i}^{*\beta_2}(\Psi_{eff})_j^{\alpha_2}\right].
\end{aligned} \tag{4.14}$$

5 Type I theory as effective theory of type IIB one

In this section we will find the effective theory, which is defined as type IIB theory on the solution of boundary conditions. Correlating it with type I theory we get relations between their background fields.

5.1 Effective Hamiltonian and Lagrangian of type IIB theory with $D5$ -brane

The initial theory is described by variables x^i , θ^{α_1} , θ^{α_2} , $\bar{\theta}^{\alpha_1}$ and $\bar{\theta}^{\alpha_2}$. On the solution of the constraints initial theory turns into effective one expressed in terms of effective variables with well defined Ω parity q^i , ξ^{α_1} , ξ^{α_2} , $\tilde{\xi}^{\alpha_1}$ and $\tilde{\xi}^{\alpha_2}$.

To obtain an effective Hamiltonian we have to put the solutions of the constraints into the expression for canonical Hamiltonian. First, we introduce effective current

$$\begin{aligned}
\tilde{I}_{\pm i} &= p_i \pm \kappa G_{ij}^{eff} q^j - 4\Re[(\Psi_{eff})_i^{\alpha_1} p_{\alpha_1} + (\Psi_{eff})_i^{\alpha_2} p_{\alpha_2}] \\
&+ 4\Re\left[\Pi_{\mp i}^j \Psi_{-j}^{\alpha_1}(p_{\alpha_1} \pm \tilde{p}_{\alpha_1}) + \Pi_{\mp i}^j \Psi_{+j}^{\alpha_2}(p_{\alpha_2} \pm \tilde{p}_{\alpha_2})\right],
\end{aligned} \tag{5.1}$$

and correlate it with the currents $I_{\pm i}$ (3.4)

$$I_{\pm i} = \pm 2(\Pi_{\pm} G_{eff}^{-1})_i^j \tilde{I}_{\pm j}. \tag{5.2}$$

Now, substituting last relation and the solutions (4.1)-(4.5) into canonical Hamiltonian (3.3), we get the expression for effective one

$$\begin{aligned}
\mathcal{H}_c^{eff} &= \tilde{t}_- - \tilde{t}_+ \\
&+ 2\Re\left[\Psi_{-i}^{\alpha_1}\left(\frac{1}{\kappa}G_{eff}^{ij}\tilde{p}_{\alpha_1} - 2\Theta^{ij}p_{\alpha_1}\right)p_j + \Psi_{+i}^{\alpha_2}\left(\frac{1}{\kappa}G_{eff}^{ij}\tilde{p}_{\alpha_2} - 2\Theta^{ij}p_{\alpha_2}\right)p_j\right] \\
&+ 2\Re\left[\left(\tilde{\xi}'^{\alpha_1} + \Psi_{-i}^{\alpha_1}q^i\right)p_{\alpha_1} + \left(\xi'^{\alpha_1} + \Psi_{+i}^{\alpha_1}q^i\right)\tilde{p}_{\alpha_1}\right] \\
&+ 2\Re\left[\left(\tilde{\xi}'^{\alpha_2} + \Psi_{+i}^{\alpha_2}q^i\right)p_{\alpha_2} + \left(\xi'^{\alpha_2} + \Psi_{-i}^{\alpha_2}q^i\right)\tilde{p}_{\alpha_2}\right] \\
&- \frac{1}{\kappa}\Re\left[p_{\alpha_1}\left((f_{11}^a)^{\alpha_1\beta_1} + 4\kappa\Psi_{-i}^{\alpha_1}\Theta^{i\beta_1}\right)p_{\beta_1}\right] + \frac{1}{\kappa}\Re\left[\tilde{p}_{\alpha_1}(f_{11}^a)^{\alpha_1\beta_1}\tilde{p}_{\beta_1}\right] \\
&- \frac{1}{\kappa}\Re\left[p_{\alpha_2}\left((f_{22}^a)^{\alpha_2\beta_2} + 4\kappa\Psi_{+i}^{\alpha_2}\Theta^{i\beta_2}\right)p_{\beta_2}\right] + \frac{1}{\kappa}\Re\left[\tilde{p}_{\alpha_2}(f_{22}^a)^{\alpha_2\beta_2}\tilde{p}_{\beta_2}\right] \\
&+ \frac{4}{\kappa}\Re\left[\tilde{p}_{\alpha_1}\Psi_{-i}^{\alpha_1}(G_{eff}^{-1})^{ij}(\Psi_{eff})_j^{\beta_1}p_{\beta_1}\right] + \frac{4}{\kappa}\Re\left[\tilde{p}_{\alpha_2}\Psi_{+i}^{\alpha_2}(G_{eff}^{-1})^{ij}(\Psi_{eff})_j^{\beta_2}p_{\beta_2}\right]
\end{aligned}$$

$$\begin{aligned}
& - \frac{1}{\kappa} \Re \left[p_{\alpha_1} \left(f_{14}^{\alpha_1 \beta_1} + 4\kappa \Psi_{-i}^{\alpha_1} \Theta^{*i \beta_1} \right) p_{\beta_1}^* \right] + \frac{1}{\kappa} \Re \left(\tilde{p}_{\alpha_1} f_{14}^{\alpha_1 \beta_1} \tilde{p}_{\beta_1}^* \right) \\
& + \frac{4}{\kappa} \Re \left[\tilde{p}_{\alpha_1} \Psi_{-i}^{\alpha_1} (G_{eff}^{-1})^{ij} (\Psi_{eff})_j^{* \beta_1} p_{\beta_1}^* \right] \\
& - \frac{1}{\kappa} \Re \left[p_{\alpha_2} \left(f_{23}^{\alpha_2 \beta_2} + 4\kappa \Psi_{+i}^{\alpha_2} \Theta^{*i \beta_2} \right) p_{\beta_2}^* \right] + \frac{1}{\kappa} \Re \left(\tilde{p}_{\alpha_2} f_{23}^{\alpha_2 \beta_2} \tilde{p}_{\beta_2}^* \right) \\
& + \frac{4}{\kappa} \Re \left[\tilde{p}_{\alpha_2} \Psi_{+i}^{\alpha_2} (G_{eff}^{-1})^{ij} (\Psi_{eff})_j^{* \beta_2} p_{\beta_2}^* \right] \\
& + \frac{4}{\kappa} \Re \left[\tilde{p}_{\alpha_1} \Psi_{-i}^{\alpha_1} (G_{eff}^{-1})^{ij} (\Psi_{eff})_j^{\beta_2} p_{\beta_2} \right] + \frac{4}{\kappa} \Re \left[\tilde{p}_{\alpha_2} \Psi_{+i}^{\alpha_2} (G_{eff}^{-1})^{ij} (\Psi_{eff})_j^{\beta_1} p_{\beta_1} \right] \\
& - \frac{1}{\kappa} \Re \left[p_{\alpha_1} \left(f_{12}^{\alpha_1 \beta_2} + 4\kappa \Psi_{-i}^{\alpha_1} \Theta^{i \beta_2} \right) p_{\beta_2} \right] - \frac{1}{\kappa} \Re \left[p_{\alpha_2} \left(f_{21}^{\alpha_2 \beta_1} + 4\kappa \Psi_{+i}^{\alpha_2} \Theta^{i \beta_1} \right) p_{\beta_1} \right] \\
& + \frac{1}{\kappa} \Re \left(\tilde{p}_{\alpha_1} f_{12}^{\alpha_1 \beta_2} \tilde{p}_{\beta_2} \right) + \frac{1}{\kappa} \Re \left(\tilde{p}_{\alpha_2} f_{21}^{\alpha_2 \beta_1} \tilde{p}_{\beta_1} \right) \\
& + \frac{4}{\kappa} \Re \left[\tilde{p}_{\alpha_2} \Psi_{+i}^{\alpha_2} (G_{eff}^{-1})^{ij} (\Psi_{eff})_j^{* \beta_2} p_{\beta_2}^* \right] + \frac{4}{\kappa} \Re \left[\tilde{p}_{\alpha_2} \Psi_{+i}^{\alpha_2} (G_{eff}^{-1})^{ij} (\Psi_{eff})_j^{* \beta_1} p_{\beta_1}^* \right] \\
& - \frac{1}{\kappa} \Re \left[p_{\alpha_1} \left(f_{13}^{\alpha_1 \beta_2} + 4\kappa \Psi_{-i}^{\alpha_1} \Theta^{*i \beta_2} \right) p_{\beta_2}^* \right] - \frac{1}{\kappa} \Re \left[p_{\alpha_2} \left(f_{24}^{\alpha_2 \beta_1} + 4\kappa \Psi_{+i}^{\alpha_2} \Theta^{*i \beta_1} \right) p_{\beta_1}^* \right] \\
& + \frac{1}{\kappa} \Re \left(\tilde{p}_{\alpha_1} f_{13}^{\alpha_1 \beta_2} \tilde{p}_{\beta_2}^* \right) + \frac{1}{\kappa} \Re \left(\tilde{p}_{\alpha_2} f_{24}^{\alpha_2 \beta_1} \tilde{p}_{\beta_1}^* \right), \tag{5.3}
\end{aligned}$$

where

$$t_{\pm} = \tilde{t}_{\pm} = \mp \frac{1}{4\kappa} (G_{eff}^{-1})^{ij} \tilde{I}_{\pm i} \tilde{I}_{\pm j}. \tag{5.4}$$

From effective Lagrangian

$$\mathcal{L}^{eff} = \dot{q}^i p_i + 2\Re \left(\dot{\xi}^{\alpha_1} p_{\alpha_1} + \dot{\xi}^{\alpha_2} p_{\alpha_2} + \dot{\xi}^{\alpha_1} \tilde{p}_{\alpha_1} + \dot{\xi}^{\alpha_2} \tilde{p}_{\alpha_2} \right) - \tilde{\mathcal{H}}_c, \tag{5.5}$$

and equations of motion for momentum p_i , we find the relation

$$p_i = \kappa G_{ij}^{eff} \dot{q}^j + 4\Re \left[(\Psi_{eff})_i^{\alpha_1} p_{\alpha_1} + (\Psi_{eff})_i^{\alpha_2} p_{\alpha_2} \right], \tag{5.6}$$

which enables us to eliminate effective bosonic momentum p_i . Putting this relation into expression for effective current $\tilde{I}_{\pm i}$ (5.1) we obtain

$$\tilde{I}_{\pm i} = \kappa G_{ij}^{eff} (\dot{q}^j \pm q'^j) + 4\Pi_{\mp i}^j \Re \left[\Psi_{-j}^{\alpha_1} (p_{\alpha_1} \pm \tilde{p}_{\alpha_1}) + \Psi_{+j}^{\alpha_2} (p_{\alpha_2} \pm \tilde{p}_{\alpha_2}) \right]. \tag{5.7}$$

Substituting last two expression into effective Lagrangian (5.5) we obtain its final form

$$\begin{aligned}
\mathcal{L}^{eff} &= \frac{\kappa}{2} G_{ij}^{eff} \eta^{mn} \partial_m q^i \partial_n q^j + 2\Re \left[\left(\dot{\xi}^{\alpha_1} - \tilde{\xi}'^{\alpha_1} + 2(\Psi_{eff})_i^{\alpha_1} \dot{q}^i \right) p_{\alpha_1} \right] \\
&+ 2\Re \left[\left(\dot{\xi}^{\alpha_1} - \xi'^{\alpha_1} - 2(\Psi_{eff})_i^{\alpha_1} q'^i \right) \tilde{p}_{\alpha_1} \right] \\
&+ 2\Re \left[\left(\dot{\xi}^{\alpha_2} - \tilde{\xi}'^{\alpha_2} + 2(\Psi_{eff})_i^{\alpha_2} \dot{q}^i \right) p_{\alpha_2} + \left(\dot{\xi}^{\alpha_2} - \xi'^{\alpha_2} - 2(\Psi_{eff})_i^{\alpha_2} q'^i \right) \tilde{p}_{\alpha_2} \right] \\
&+ \frac{1}{\kappa} \Re \left[p_{\alpha_1} (f_{11}^{eff})^{\alpha_1 \beta_1} p_{\beta_1} - \tilde{p}_{\alpha_1} (f_{11}^{eff})^{\alpha_1 \beta_1} \tilde{p}_{\beta_1} + p_{\alpha_2} (f_{22}^{eff})^{\alpha_2 \beta_2} p_{\beta_2} - \tilde{p}_{\alpha_2} (f_{22}^{eff})^{\alpha_2 \beta_2} \tilde{p}_{\beta_2} \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{2}{\kappa} \Re \left[p_{\alpha_1} (f_{12}^{eff})^{\alpha_1 \beta_2} p_{\beta_2} - \tilde{p}_{\alpha_1} (f_{12}^{eff})^{\alpha_1 \beta_2} \tilde{p}_{\beta_2} \right] \\
& + \frac{1}{\kappa} \Re \left[p_{\alpha_1} \bar{f}_{14}^{\alpha_1 \beta_1} p_{\beta_1}^* - \tilde{p}_{\alpha_1} \bar{f}_{14}^{\alpha_1 \beta_1} \tilde{p}_{\beta_1}^* + p_{\alpha_2} \bar{f}_{23}^{\alpha_2 \beta_2} p_{\beta_2}^* - \tilde{p}_{\alpha_2} \bar{f}_{23}^{\alpha_2 \beta_2} \tilde{p}_{\beta_2}^* \right] \\
& + \frac{2}{\kappa} \Re \left[p_{\alpha_1} \bar{f}_{13}^{\alpha_1 \beta_2} p_{\beta_2}^* - \tilde{p}_{\alpha_1} \bar{f}_{13}^{\alpha_1 \beta_2} \tilde{p}_{\beta_2}^* \right], \tag{5.8}
\end{aligned}$$

where effective background fields G_{ij}^{eff} , $(\Psi_{eff})_i^{\alpha_1}$, $(\Psi_{eff})_i^{\alpha_2}$, $(f_{11}^{eff})^{\alpha_1 \beta_1}$, $(f_{12}^{eff})^{\alpha_1 \beta_2}$ and $(f_{22}^{eff})^{\alpha_2 \beta_2}$ are defined in Eq.(3.27). We also introduce notation for improved $D5$ -brane bispinors

$$\bar{f}_{14}^{\alpha_1 \beta_1} = \frac{1}{2} (f_{14}^{\alpha_1 \beta_1} - f_{14}^{* \beta_1 \alpha_1}) - \Psi_{-i}^{\alpha_1} G^{ij} \Psi_{-j}^{* \beta_1}, \quad \bar{f}_{23}^{\alpha_2 \beta_2} = \frac{1}{2} (f_{23}^{\alpha_2 \beta_2} - f_{23}^{* \beta_2 \alpha_2}) - \Psi_{+i}^{\alpha_2} G^{ij} \Psi_{+j}^{* \beta_2}, \tag{5.9}$$

$$\bar{f}_{13}^{\alpha_1 \beta_2} = \frac{1}{2} (f_{13}^{\alpha_1 \beta_2} - f_{24}^{* \beta_2 \alpha_1}) - \Psi_{-i}^{\alpha_1} G^{ij} \Psi_{+j}^{* \beta_2}, \tag{5.10}$$

which have similar forms as effective ones (3.27), but which do not appear in algebra of constraints (3.25). Note that all terms in \mathcal{L}^{eff} are Ω even.

In the supersymmetric case $\Psi_{-i}^{\alpha_1}$ and $\Psi_{+i}^{\alpha_2}$ play the same role as the antisymmetric field B_{ij} in pure bosonic case. In fact, none of them appears explicitly, but they contribute as the bilinear terms in the background fields of effective theory (3.27), (5.9) and (5.10).

Initial Lagrangian (2.11) depends on variables x^i , θ^{α_1} , θ^{α_2} , $\bar{\theta}^{\alpha_1}$, $\bar{\theta}^{\alpha_2}$, π_{α_1} , π_{α_2} , $\bar{\pi}_{\alpha_1}$ and $\bar{\pi}_{\alpha_2}$, and effective Lagrangian depends on the effective variables with well defined Ω parity q^i , ξ^{α_1} , ξ^{α_2} , $\tilde{\xi}^{\alpha_1}$, $\tilde{\xi}^{\alpha_2}$, p_{α_1} , p_{α_2} , \tilde{p}_{α_1} and \tilde{p}_{α_2} . In order to compare initial and effective theories we have to make correspondence between variables. If we substitute initial variables x^i , θ^{α_1} , θ^{α_2} , $\bar{\theta}^{\alpha_1}$ and $\bar{\theta}^{\alpha_2}$ with momenta independent parts of their solutions, $x^i \rightarrow q^i$, $\theta^{\alpha_1} \rightarrow \frac{1}{2}(\xi^{\alpha_1} + \tilde{\xi}^{\alpha_1})$, $\theta^{\alpha_2} \rightarrow \frac{1}{2}(\xi^{\alpha_2} + \tilde{\xi}^{\alpha_2})$, $\bar{\theta}^{\alpha_1} \rightarrow \frac{1}{2}(\xi^{\alpha_1} - \tilde{\xi}^{\alpha_1})$, $\bar{\theta}^{\alpha_2} \rightarrow -\frac{1}{2}(\xi^{\alpha_2} - \tilde{\xi}^{\alpha_2})$, and fermionic momenta with their complete solutions (4.2)-(4.5), $\pi_{\alpha_1} \rightarrow p_{\alpha_1} + \tilde{p}_{\alpha_1}$, $\pi_{\alpha_2} \rightarrow p_{\alpha_2} + \tilde{p}_{\alpha_2}$, $\bar{\pi}_{\alpha_1} \rightarrow p_{\alpha_1} - \tilde{p}_{\alpha_1}$ and $\bar{\pi}_{\alpha_2} \rightarrow -p_{\alpha_2} + \tilde{p}_{\alpha_2}$ in Lagrangian (2.11), we obtain

$$\begin{aligned}
\mathcal{L} \rightarrow & \frac{\kappa}{2} G_{ij} \eta^{mn} \partial_m q^i \partial_n q^j + \kappa \varepsilon^{mn} B_{ij} \partial_m q^i \partial_n q^j \\
& + 2\Re \left[\left(\dot{\xi}^{\alpha_1} - \tilde{\xi}^{\alpha_1} + \Psi_{+i}^{\alpha_1} \dot{q}^i - \Psi_{-i}^{\alpha_1} q'^i \right) p_{\alpha_1} + \left(\dot{\xi}^{\alpha_1} - \xi^{\alpha_1} + \Psi_{-i}^{\alpha_1} \dot{q}^i - \Psi_{+i}^{\alpha_1} q'^i \right) \tilde{p}_{\alpha_1} \right] \\
& + 2\Re \left[\left(\dot{\xi}^{\alpha_2} - \tilde{\xi}^{\alpha_2} + \Psi_{-i}^{\alpha_2} \dot{q}^i - \Psi_{+i}^{\alpha_2} q'^i \right) p_{\alpha_2} + \left(\dot{\xi}^{\alpha_2} - \xi^{\alpha_2} + \Psi_{+i}^{\alpha_2} \dot{q}^i - \Psi_{-i}^{\alpha_2} q'^i \right) \tilde{p}_{\alpha_2} \right] \\
& + \frac{1}{\kappa} \Re \left[p_{\alpha_1} (f_{11}^a)^{\alpha_1 \beta_1} p_{\beta_1} - \tilde{p}_{\alpha_1} (f_{11}^a)^{\alpha_1 \beta_1} \tilde{p}_{\beta_1} + p_{\alpha_2} (f_{22}^a)^{\alpha_2 \beta_2} p_{\beta_2} - \tilde{p}_{\alpha_2} (f_{22}^a)^{\alpha_2 \beta_2} \tilde{p}_{\beta_2} \right] \\
& + \frac{2}{\kappa} \Re \left[\tilde{p}_{\alpha_1} (f_{11}^s)^{\alpha_1 \beta_1} p_{\beta_1} + \tilde{p}_{\alpha_2} (f_{22}^s)^{\alpha_2 \beta_2} p_{\beta_2} \right] \\
& + \frac{2}{\kappa} \Re \left[p_{\alpha_1} \frac{1}{2} (f_{12}^{\alpha_1 \beta_2} - f_{21}^{\beta_2 \alpha_1}) p_{\beta_2} - \tilde{p}_{\alpha_1} \frac{1}{2} (f_{12}^{\alpha_1 \beta_2} - f_{21}^{\beta_2 \alpha_1}) \tilde{p}_{\beta_2} \right] \\
& + \frac{1}{\kappa} \Re \left[\tilde{p}_{\alpha_1} (f_{12}^{\alpha_1 \beta_2} + f_{21}^{\beta_2 \alpha_1}) p_{\beta_2} - p_{\alpha_1} (f_{12}^{\alpha_1 \beta_2} + f_{21}^{\beta_2 \alpha_1}) \tilde{p}_{\beta_2} \right]
\end{aligned}$$

$$\begin{aligned}
& + \frac{1}{\kappa} \Re \left[p_{\alpha_1} \frac{1}{2} (f_{14}^{\alpha_1 \beta_1} - f_{14}^{* \beta_1 \alpha_1}) p_{\beta_1}^* - \tilde{p}_{\alpha_1} \frac{1}{2} (f_{14}^{\alpha_1 \beta_1} - f_{14}^{* \beta_1 \alpha_1}) \tilde{p}_{\beta_1}^* \right] \\
& + \frac{1}{\kappa} \Re \left[p_{\alpha_2} \frac{1}{2} (f_{23}^{\alpha_2 \beta_2} - f_{23}^{* \beta_2 \alpha_2}) p_{\beta_2}^* - \tilde{p}_{\alpha_2} \frac{1}{2} (f_{23}^{\alpha_2 \beta_2} - f_{23}^{* \beta_2 \alpha_2}) \tilde{p}_{\beta_2}^* \right] \\
& - \frac{1}{\kappa} \Re \left[p_{\alpha_1} (f_{14}^{\alpha_1 \beta_1} + f_{14}^{* \beta_1 \alpha_1}) \tilde{p}_{\beta_1}^* + p_{\alpha_2} (f_{23}^{\alpha_2 \beta_2} + f_{23}^{* \beta_2 \alpha_2}) \tilde{p}_{\beta_2}^* \right] \\
& + \frac{2}{\kappa} \Re \left[p_{\alpha_1} \frac{1}{2} (f_{13}^{\alpha_1 \beta_2} - f_{24}^{* \beta_2 \alpha_1}) p_{\beta_2}^* - \tilde{p}_{\alpha_1} \frac{1}{2} (f_{13}^{\alpha_1 \beta_2} - f_{24}^{* \beta_2 \alpha_1}) \tilde{p}_{\beta_2}^* \right] \\
& + \frac{1}{\kappa} \Re \left[\tilde{p}_{\alpha_1} (f_{13}^{\alpha_1 \beta_2} + f_{24}^{* \beta_2 \alpha_1}) p_{\beta_2}^* - p_{\alpha_1} (f_{13}^{\alpha_1 \beta_2} + f_{24}^{* \beta_2 \alpha_1}) \tilde{p}_{\beta_2}^* \right]. \tag{5.11}
\end{aligned}$$

Comparing this Lagrangian with effective one (5.8) we can conclude that $\mathcal{L} \rightarrow \mathcal{L}^{eff}$ after substitution

$$\begin{aligned}
G_{ij} & \rightarrow G_{ij}^{eff}, & B_{ij} & \rightarrow 0, \\
\frac{1}{2} \Psi_{+i}^{\alpha_1} & \rightarrow (\Psi_{eff})_i^{\alpha_1}, & \frac{1}{2} \Psi_{-i}^{\alpha_2} & \rightarrow (\Psi_{eff})_i^{\alpha_2}, & \Psi_{-i}^{\alpha_1} & \rightarrow 0, & \Psi_{+i}^{\alpha_2} & \rightarrow 0, \\
(f_{11}^a)^{\alpha_1 \beta_1} & \rightarrow (f_{11}^{eff})^{\alpha_1 \beta_1}, & (f_{22}^a)^{\alpha_2 \beta_2} & \rightarrow (f_{22}^{eff})^{\alpha_2 \beta_2}, & (f_{11}^s)^{\alpha_1 \beta_1} & \rightarrow 0, & (f_{22}^s)^{\alpha_2 \beta_2} & \rightarrow 0, \\
\frac{1}{2} (f_{12}^{\alpha_1 \beta_2} - f_{21}^{\beta_2 \alpha_1}) & \rightarrow (f_{12}^{eff})^{\alpha_1 \beta_2}, & & & f_{12}^{\alpha_1 \beta_2} + f_{21}^{\beta_2 \alpha_1} & \rightarrow 0, \\
\frac{1}{2} (f_{14}^{\alpha_1 \beta_1} - f_{14}^{* \beta_1 \alpha_1}) & \rightarrow \bar{f}_{14}^{\alpha_1 \beta_1}, & & & f_{14}^{\alpha_1 \beta_1} + f_{14}^{* \beta_1 \alpha_1} & \rightarrow 0, \\
\frac{1}{2} (f_{23}^{\alpha_2 \beta_2} - f_{23}^{* \beta_2 \alpha_2}) & \rightarrow \bar{f}_{23}^{\alpha_2 \beta_2}, & & & f_{23}^{\alpha_2 \beta_2} + f_{23}^{* \beta_2 \alpha_2} & \rightarrow 0, \\
\frac{1}{2} (f_{13}^{\alpha_1 \beta_2} - f_{24}^{* \beta_2 \alpha_1}) & \rightarrow \bar{f}_{13}^{\alpha_1 \beta_2}, & & & f_{13}^{\alpha_1 \beta_2} + f_{24}^{* \beta_2 \alpha_1} & \rightarrow 0. \tag{5.12}
\end{aligned}$$

In our approach the effective theory has been obtained from initial IIB one with embedded $D5$ -brane on the solution of boundary conditions. The condition of 2π periodicity of all canonical variables solves boundary condition at $\sigma = \pi$ in terms of that at $\sigma = 0$, and consequently, this is a closed string theory. The effective theory is Ω symmetric part of type IIB superstring theory, and we expect that it should correspond to the type I superstring theory with embedded $D5$ -brane, which will be shown in the next subsection.

5.2 Type I superstring theory with $D5$ -brane

In order to work with stable initial and final theories we should embed $D5$ -brane both in type IIB and type I string theories. We have already done this for type IIB theory and now we will apply similar procedure for type I superstring theory. The corresponding Lagrangian is of the form [1]

$$\begin{aligned}
\mathcal{L}^I & = \frac{\kappa}{2} G_{\mu\nu}^I \eta^{mn} \partial_m q^\mu \partial_n q^\nu + \\
& - \pi_\alpha (\partial_\tau - \partial_\sigma) [\eta^\alpha + (\Psi_I)_\mu^\alpha q^\mu] + (\partial_\tau + \partial_\sigma) [\bar{\eta}^\alpha + (\Psi_I)_\mu^\alpha q^\mu] \bar{\pi}_\alpha - \frac{1}{2\kappa} \pi_\alpha (F_I^* \Gamma)^{\alpha\beta} \bar{\pi}_\beta. \tag{5.13}
\end{aligned}$$

Index I stands for type I superstring theory and variable q^μ is symmetric under Ω transformation. The field strength $(F_I^* \Gamma)^{\alpha\beta}$ is antisymmetric under permutation of indices, $(F_I^* \Gamma)^{\alpha\beta} = -(F_I^* \Gamma)^{\beta\alpha}$, where matrix $^* \Gamma$ is defined as (see Appendix A)

$$^* \Gamma \equiv \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^4 \Gamma^5 = \begin{pmatrix} -\gamma & 0 & 0 & 0 \\ 0 & -\gamma & 0 & 0 \\ 0 & 0 & -\gamma & 0 \\ 0 & 0 & 0 & -\gamma \end{pmatrix}. \quad (5.14)$$

Let us embed $D5$ -brane in this theory. For coordinates q^i ($i = 0, 1, \dots, 5$) we choose Neumann and for q^a ($a = 6, 7, 8, 9$) Dirichlet boundary conditions. We can take that q^i and q^a are orthogonal coordinates which implies $G_{ia}^I = 0$. Also we take that field $(\Psi_I)_\mu^\alpha$ is nontrivial only on $D5$ -brane, $(\Psi_I)_\mu^\alpha \rightarrow (\Psi_I)_i^\alpha$. The term which describes the free string oscillation in q^a directions decouples from the rest.

In Table 2 we summarize the list of the background fields of the effective theory in $D = 10$ dimensional space-time, fields living on the $D5$ -brane and the rest fields.

Sector	x^μ ($\mu = 0, 1, \dots, 9$)	x^i ($i = 0, 1, \dots, 5$)	x^a ($a = 6, 7, 8, 9$)
NS-NS	$G_{\mu\nu}^{eff}$ $\Phi = 0$	G_{ij}^{eff}	G_{ab}^{eff} (decoupled)
NS-R	$(\Psi_{eff})_\mu^\alpha$	$(\Psi_{eff})_i^{\alpha_1}, (\Psi_{eff})_i^{\alpha_2}$	
R-R	$F_{eff}^{\alpha\beta}$	$(f_{11}^{eff})^{\alpha_1\beta_1}, (f_{22}^{eff})^{\alpha_2\beta_2}, (f_{12}^{eff})^{\alpha_1\beta_2}$ $\bar{f}_{14}^{\alpha_1\beta_1}, \bar{f}_{23}^{\alpha_2\beta_2}, \bar{f}_{13}^{\alpha_1\beta_2}$	

Table 2: Background fields of the effective theory: the complete set, part living on $D5$ -brane and the rest fields eliminated from the theory

Now, using the relation (A.16) and (B.8), we rewrite the Lagrangian (5.13) in terms of $D5$ -brane variables and background fields. We want to establish relation of such theory with effective one (5.8). Comparing terms linear in fermionic momenta and independent of background fields, we obtain the following connection between coordinates and momenta of these theories

$$\begin{aligned} \eta^\alpha &= \frac{1}{2}(\xi^\alpha + \tilde{\xi}^\alpha), \quad \bar{\eta}^\alpha = -\frac{1}{2} \left[^* \Gamma (\xi - \tilde{\xi}) \right]^\alpha, \\ p_\alpha &= p_\alpha + \tilde{p}_\alpha, \quad \bar{\pi}_\alpha = - \left[^* \Gamma (p - \tilde{p}) \right]_\alpha, \end{aligned} \quad (5.15)$$

where

$$\xi^\alpha = P_s \theta^\alpha = -P_s (^* \Gamma \bar{\theta})^\alpha, \quad \tilde{\xi}^\alpha = P_a [\theta^\alpha + (^* \Gamma \bar{\theta})^\alpha], \quad (5.16)$$

$$p_\alpha = P_s \pi_\alpha = -P_s (\bar{\pi} ^* \Gamma)_\alpha, \quad \tilde{p}_\alpha = P_a \pi_\alpha = P_a (\bar{\pi} ^* \Gamma)_\alpha. \quad (5.17)$$

Note that these are just relations (4.6) rewritten in terms of ten dimensional spinors. Putting these relations into expression for \mathcal{L}^I , we have

$$\mathcal{L}^I(\mathcal{A}^I) = \mathcal{L}^{eff}(\mathcal{A}^{eff}), \quad (5.18)$$

where \mathcal{A}^I and \mathcal{A}^{eff} denote background fields of type I superstring theory and the effective theory, respectively. So, $D5$ -brane background fields of type I theory can be expressed in terms of the corresponding ones of type IIB theory as

$$G_{ij}^I = G_{ij}^{eff}, \quad (\Psi_I)_i^{\alpha_1} = (\Psi_{eff})_i^{\alpha_1}, \quad (\Psi_I)_i^{\alpha_2} = (\Psi_{eff})_i^{\alpha_2}, \quad (5.19)$$

$$(f_{11}^{a(I)})^{\alpha_1\beta_1} = (f_{11}^{eff})^{\alpha_1\beta_1}, \quad (f_{22}^{a(I)})^{\alpha_2\beta_2} = (f_{22}^{eff})^{\alpha_2\beta_2}, \quad (5.20)$$

$$\frac{1}{2}(f_{12}^{(I)\alpha_1\beta_2} - f_{21}^{(I)\beta_2\alpha_1}) = (f_{12}^{eff})^{\alpha_1\beta_2}, \quad (5.21)$$

$$\begin{aligned} \frac{1}{2}(f_{14}^{(I)\alpha_1\beta_1} - f_{14}^{*(I)\beta_1\alpha_1}) &= \bar{f}_{14}^{\alpha_1\beta_1}, & \frac{1}{2}(f_{23}^{(I)\alpha_2\beta_2} - f_{23}^{*(I)\beta_2\alpha_2}) &= \bar{f}_{23}^{\alpha_2\beta_2}, \\ \frac{1}{2}(f_{13}^{(I)\alpha_1\beta_2} - f_{24}^{*(I)\beta_2\alpha_1}) &= \bar{f}_{13}^{\alpha_1\beta_2}, \end{aligned}$$

where the right-hand sides are defined in (3.27), (5.9) and (5.10). The bispinors from the last three lines in Eq.(5.21) can be written in terms of antisymmetric tensors (see Appendix B)

$$\begin{aligned} f_{(0)}^{(I)} &= f_{(0)} - \frac{1}{6}G^{ij}\Psi_{\pm i}\Psi_{\pm j}, & f_{ij}^{(I)} &= f_{ij} - \frac{1}{6}G^{kl}\Psi_{\pm k}\gamma_{[ij]}\Psi_{\pm l}, \\ f_i^{(I)} &= f_i - \frac{1}{6}G^{kl}\Psi_{\pm k}\gamma_i\Psi_{\pm l}, & f_{ijk}^{(I)} &= f_{ijk} - \frac{1}{6}G^{lm}\Psi_{\pm l}\gamma_{[ijk]}\Psi_{\pm m}. \end{aligned} \quad (5.22)$$

Here we write general forms of type I $D5$ -brane R-R background fields in terms of type IIB ones. Field strength f_i and f_{ijk} appear as the coefficients in gamma matrix expansion of the bispinors f_{11} , f_{22} , f_{14} and f_{23} , while $f_{(0)}$ and f_{ij} in the same sense are related to f_{12} , f_{21} , f_{13} and f_{24} . Because $f_{11}^{a(I)}$ and $f_{22}^{a(I)}$ are antisymmetric tensors, they can be expressed in terms of f_i , while $f_{14}^{(I)}$ and $f_{23}^{(I)}$ contain both f_i and f_{ijk} tensors.

Fields B_{ij} , $\Psi_{-i}^{\alpha_1}$ and $\Psi_{+i}^{\alpha_2}$, odd under Ω parity transformation, are not completely eliminated. They appear as bilinear terms in type I $D5$ -brane background fields. Consequently, we obtained the generalized expressions for type I $D5$ -brane background fields in terms of the type IIB $D5$ -brane background fields. The quadratic terms in the expressions for effective background fields can be considered as supersymmetric generalization of the open string metric $G_{\mu\nu}^{eff}$ obtained by Seiberg and Witten [8].

5.3 Embedding of $D5$ -brane in type IIB and type I superstring theory using T-duality

Using the expressions for type I superstring background fields in terms of type IIB ones obtained in [1] and T-duality transformations [9] along x^a ($a = 6, 7, 8, 9$) directions, we find the expressions for background fields obtained in previous subsection.

In the third article of [9] type IIA/B superstring theory is considered and finding of T-duality transformation along one direction is demonstrated. Here we will apply the presented procedure for type IIB superstring theory and four x^a directions which are chosen to be orthogonal to $D5$ -brane.

We start with the action of type IIB superstring theory in pure spinor formulation given in (2.10) in the form

$$S = \kappa \int_{\Sigma} d^2 \xi \partial_+ x^\mu \Pi_{+\mu\nu} \partial_- x^\nu + \int_{\Sigma} d^2 \xi \left[-\pi_\alpha \partial_- (\theta^\alpha + \Psi_\mu^\alpha x^\mu) + \partial_+ (\bar{\theta}^\alpha + \bar{\Psi}_\mu^\alpha x^\mu) \bar{\pi}_\alpha + \frac{1}{2\kappa} \pi_\alpha F^{\alpha\beta} \bar{\pi}_\beta \right], \quad (5.23)$$

where the background fields are constant, $\Pi_{\pm\mu\nu} = B_{\mu\nu} \pm \frac{1}{2} G_{\mu\nu}$ and $\partial_\pm = \partial_\tau \pm \partial_\sigma$. We suppose that the action has a global shift symmetry in x^a ($a = 6, 7, 8, 9$) directions. So, we have to introduce gauge fields v_\pm^a and make a change in the action

$$\partial_\pm x^a \rightarrow \partial_\pm x^a + v_\pm^a. \quad (5.24)$$

For the fields v_\pm^a we have to introduce additional term in the action

$$S_{gauge} = \frac{1}{2} \kappa \int_{\Sigma} d^2 \xi y_a (\partial_+ v_-^a - \partial_- v_+^a), \quad (5.25)$$

which produces vanishing of the field strength $\partial_+ v_-^a - \partial_- v_+^a$ if we vary the action with respect to the Lagrange multipliers y_a . The full action has the form

$$S^* = S(x^i, x^a, v_\pm^a) + S_{gauge}(y^a, v_\pm^a). \quad (5.26)$$

Let us note that on the equations of motion for y_a we have $v_\pm^a = \partial_\pm x^a$ and the original dynamics survives unchanged.

Now we can fix x^a to zero and obtain the action quadratic in the fields v_\pm^a , which can be integrated out classically. On the equations of motion for v_\pm^a we obtain expressions for these gauge fields in terms of x^i ($i = 0, 1, \dots, 5$), y_a and momenta, π_α and $\bar{\pi}_\alpha$,

$$v_+^a = 2(2\partial_+ x^i \Pi_{+ib} - \partial_+ y_b - \frac{2}{\kappa} \pi_\alpha \Psi_b^\alpha) \Theta_-^{ba}, \quad (5.27)$$

$$v_-^a = 2\Theta_-^{ab} (2\Pi_{+bi} \partial_- x^i + \partial_- y_b + \frac{2}{\kappa} \bar{\Psi}_b^\alpha \bar{\pi}_\alpha), \quad (5.28)$$

where

$$\Theta_-^{ab} = (G^{-1}\Pi_-g_{eff}^{-1})^{ab}, \quad g_{ab}^{eff} \equiv -4\Pi_{\pm ac}G^{cd}\Pi_{\mp db} = G_{ab} - 4B_{ac}G^{cd}B_{db}. \quad (5.29)$$

Substituting expression for v_{\pm}^a in the action S^* we obtain the the dual action from which we read the dual background fields

$$\tilde{\Pi}_{+ij} = \Pi_{+ij} + 4\Pi_{+ia}\Theta_-^{ab}\Pi_{+bj}, \quad (5.30)$$

$$\tilde{\Pi}_{+i}{}^a = 2\Pi_{+ib}\Theta_-^{ba}, \quad \tilde{\Pi}^a{}_{+i} = -2\Theta_-^{ab}\Pi_{+bi}, \quad (5.31)$$

$$\tilde{\Pi}_+^{ab} = -\Theta_-^{ab}, \quad (5.32)$$

$$\tilde{\Psi}_i^\alpha = \Psi_i^\alpha + 4\Psi_a^\alpha\Theta_-^{ab}\Pi_{+bi}, \quad (5.33)$$

$$\tilde{\tilde{\Psi}}_i^\alpha = -(*\Gamma\bar{\Psi})_i^\alpha - 4\Pi_{+ia}\Theta_-^{ab}(*\Gamma\bar{\Psi})_b^\alpha, \quad (5.34)$$

$$\tilde{\Psi}^{\alpha a} = 2\Psi_b^\alpha\Theta_-^{ba}, \quad (5.35)$$

$$\tilde{\tilde{\Psi}}^{\alpha a} = 2\Theta_-^{ab}(*\Gamma\bar{\Psi})_b^\alpha, \quad (5.36)$$

$$\tilde{F}^{\alpha\beta} = -\left[(F - 8\Psi_a\Theta_-^{ab}\bar{\Psi}_b)*\Gamma\right]^{\alpha\beta}. \quad (5.37)$$

Symbol \tilde{A} denotes dual background field of the arbitrary field A . We redefine the fields $\bar{\Psi}_\mu^\alpha$ and $F^{\alpha\beta}$ as well as all other bar variables multiplying them with $-*\Gamma$ (5.14). From (5.30)-(5.32) we obtain the T-duality transformation rules for the fields $G_{\mu\nu}$ and $B_{\mu\nu}$

$$\begin{aligned} \tilde{G}_{ij} &= G_{ij} + 4G_{ia}\Theta^{ab}B_{bj} + 4B_{ia}\Theta^{ab}G_{bj} - 4B_{ia}(g_{eff}^{-1})^{ab}B_{bj} - G_{ia}(g_{eff}^{-1})^{ab}G_{bj}, \\ \tilde{G}_i{}^a &= 2G_{ib}\Theta^{ba} - 2B_{ib}(g_{eff}^{-1})^{ba}, \quad \tilde{G}^{ab} = (g_{eff}^{-1})^{ab}, \end{aligned} \quad (5.38)$$

$$\begin{aligned} \tilde{B}_{ij} &= B_{ij} + 4B_{ia}\Theta^{ab}B_{bj} + G_{ia}\Theta^{ab}G_{bj} - G_{ia}\Theta^{ab}B_{bj} - B_{ia}(g_{eff}^{-1})^{ab}G_{bj}, \\ \tilde{B}_i{}^a &= 2B_{ib}\Theta^{ba} - \frac{1}{2}G_{ib}(g_{eff}^{-1})^{ba}, \quad \tilde{B}^{ab} = -\Theta^{ab}, \end{aligned} \quad (5.39)$$

where $\Theta^{ab} = (g_{eff}^{-1}BG^{-1})^{ab}$.

Our choice of background fields introduced in Section 2, $G_{ia} = 0$, $B_{ia} = B_{ab} = 0$, $\Psi_a^\alpha = \bar{\Psi}_a^\alpha = 0$, implies

$$\tilde{G}_{ij} = G_{ij}, \quad \tilde{G}_i{}^a = 0, \quad \tilde{G}^{ab} = G^{ab}, \quad (5.40)$$

$$\tilde{B}_{ij} = B_{ij}, \quad \tilde{B}_i{}^a = 0, \quad \tilde{B}^{ab} = 0, \quad (5.41)$$

$$\tilde{\Psi}_i^{\alpha 1} = \Psi_i^{\alpha 1}, \quad \tilde{\Psi}_i^{\alpha 2} = \Psi_i^{\alpha 2}, \quad \tilde{\tilde{\Psi}}_i^{\alpha 1} = \tilde{\Psi}_i^{*\alpha 1}, \quad \tilde{\tilde{\Psi}}_i^{\alpha 2} = -\bar{\Psi}_i^{\alpha 2}, \quad (5.42)$$

$$\tilde{f}_{11}^{\alpha 1\beta 1} = f_{11}^{\alpha 1\beta 1}, \quad \tilde{f}_{12}^{\alpha 1\beta 2} = -f_{12}^{\alpha 1\beta 2}, \quad \tilde{f}_{21}^{\alpha 2\beta 1} = f_{21}^{\alpha 2\beta 1}, \quad \tilde{f}_{22}^{\alpha 2\beta 2} = -f_{22}^{\alpha 2\beta 2}, \quad (5.43)$$

$$\tilde{f}_{13}^{\alpha 1\beta 2} = -f_{13}^{\alpha 1\beta 2}, \quad \tilde{f}_{14}^{\alpha 1\beta 1} = f_{14}^{\alpha 1\beta 1}, \quad \tilde{f}_{23}^{\alpha 2\beta 2} = -f_{23}^{\alpha 2\beta 2}, \quad \tilde{f}_{24}^{\alpha 2\beta 1} = f_{24}^{\alpha 2\beta 1}. \quad (5.44)$$

The same procedure we repeat for type I superstring theory which Lagrangian is given in (5.13). The form of the dual field for type I superstring theory is the same as for type

IIB one up to the condition $B_{\mu\nu} = 0$ and $\Psi_\mu^\alpha = \bar{\Psi}_\mu^\alpha$. We have

$$\tilde{G}_{ij}^I = G_{ij}^I - G_{ia}^I G_I^{ab} G_{bj}^I, \quad (\tilde{G}^I)_i{}^a = 0, \quad \tilde{G}_I^{ab} = G_I^{ab}, \quad (5.45)$$

$$\tilde{B}_{ij}^I = 0, \quad (\tilde{B}^I)_i{}^a = -\frac{1}{2} G_{ib}^I G_I^{ba}, \quad \tilde{B}_I^{ab} = 0, \quad (5.46)$$

$$(\tilde{\Psi}_I)_i^\alpha = (\Psi_I)_i^\alpha - (\Psi_I)_a^\alpha (G_I)^{ab} G_{bi}^I, \quad (5.47)$$

$$(\tilde{\Psi}_I)^{\alpha a} = -(\Psi_I)_b^\alpha G_I^{ba}, \quad (5.48)$$

$$\tilde{F}_I^{\alpha\beta} = F_I^{\alpha\beta} + 4(\Psi_I)_a^\alpha G_I^{ab} (\Psi_I)_b^\beta, \quad (5.49)$$

where for simlicity $F_I^{\alpha\beta}$ stands instead $-(F_I^* \Gamma)^{\alpha\beta}$ used in Eq.(5.13). Choosing background fields as in Section 2 of the present paper, we have

$$\tilde{G}_{ij}^I = G_{ij}^I, \quad (\tilde{G}^I)_i{}^a = 0, \quad \tilde{G}_I^{ab} = G_I^{ab}, \quad (5.50)$$

$$(\tilde{\Psi}_I)_\mu^\alpha = (\Psi_I)_\mu^\alpha, \quad \tilde{F}_I^{\alpha\beta} = F_I^{\alpha\beta}, \quad (5.51)$$

$$\tilde{B}_{ij}^I = (\tilde{B}_I)_i{}^a = (\tilde{B}_I)^{ab} = 0, \quad (\tilde{\Psi})^{\alpha a} = 0. \quad (5.52)$$

Using obtained duality transformation for type IIB and type I superstring theories we can reproduct the results of the prsent paper from the results of Ref.[1]. For example, from the expression for type I superstring metric obtained in [1]

$$G_{\mu\nu}^I = G_{\mu\nu} - 4B_{\mu\rho} G^{\rho\lambda} B_{\lambda\nu}, \quad (5.53)$$

after T-duality transformation we have

$$\tilde{G}_{ij}^I = \tilde{G}_{ij} - 4\tilde{B}_{ik} \tilde{G}^{kl} \tilde{B}_{lj} - 4\tilde{B}_i{}^a \tilde{B}_{aj} = G_{ij}^{eff} = G_{ij}^I. \quad (5.54)$$

6 Concluding remarks

In this paper we considered relation between type IIB and type I theories with embedded $D5$ -branes. We used the pure spinor formulation of the theories introduced in Refs.[5] restricting our analysis to the quadratic terms. We suppose that all background fields, the metric tensor $G_{\mu\nu}$, antisymmetric NS-NS field $B_{\mu\nu}$, gravitino fields Ψ_μ^α and $\bar{\Psi}_\mu^\alpha$, and the R-R field strength $F^{\alpha\beta}$, are constant.

In Ref.[1] we showed that type IIB superstring theory, on the solution of appropriately chosen open string boundary conditions, corresponds to the type I superstring theory. It means that we obtained relation between $D9$ -branes in these theories. In the present article, instead $D9$ -brane we considered $D5$ -branes which are stable in both theories.

Using canonical method, following [10], we derived boundary conditions from the requirement that Hamiltonian, as time translation generator, has well defined functional

derivatives in supercoordinates and their canonically conjugated supermomenta. All boundary conditions at string endpoints we treated as canonical constraints. Applying Dirac consistency procedure they produced an infinite set of constraints. With the help of Taylor expansion they can be rewritten as five σ -dependent constraints. All these constraints, originating from boundary conditions, are of the second class and we can solve them. We obtained the expressions for coordinates x^i , θ^{α_1} , $\bar{\theta}^{\alpha_1}$, θ^{α_2} and $\bar{\theta}^{\alpha_2}$ in terms of effective ones, q^i , ξ^{α_1} , $\tilde{\xi}^{\alpha_1}$, ξ^{α_2} and $\tilde{\xi}^{\alpha_2}$ (momenta independent parts of the solutions for initial supercoordinates x^i , θ^{α_1} , $\bar{\theta}^{\alpha_1}$, θ^{α_2} and $\bar{\theta}^{\alpha_2}$) and momenta p_i , p_{α_1} and p_{α_2} (canonically conjugated to q^i , ξ^{α_1} and ξ^{α_2} respectively).

Effective Lagrangian, obtained on the solution of boundary conditions, is even under orientifold projection Ω in six dimensions. In fact it has form of type I superstring theory with embedded $D5$ -brane. As a result we obtained the expressions for $D5$ -brane background fields of type I theory in terms of the corresponding ones of type IIB (5.21). Note that second parts of effective backgrounds in (3.27), (5.9) and (5.10), bilinear in Ω odd fields are our improvements to the well known first parts, linear in Ω even fields. Seiberg and Witten [8] obtained term with square of the Kalb-Ramond field $B_{\mu\nu}$ in the open string metric $G_{\mu\nu}^{eff}$. Our quadratic parts of the effective background fields can be considered as a supersymmetric generalization of their result.

In subsection 5.3 we showed that there is a relation between the results of the present paper and those from [1]. This connection is realized by T-duality transformations along the x^a directions, which are orthogonal to $D5$ -brane.

Table 3 contains the background fields of the type IIB superstring theory with embedded $D5$ -brane, Ω even projection of type IIB with $D5$ -brane, effective theory and type I with $D5$ -brane.

Theory	NS-NS	NS-R	R-R
Type IIB	$G_{ij}, B_{ij}, \Phi(=0)$	$\Psi_i^{\alpha_1}, \Psi_i^{\alpha_2}$ $\bar{\Psi}_i^{\alpha_1}, \bar{\Psi}_i^{\alpha_2}$	$f_{11}^{\alpha_1\beta_1}, f_{12}^{\alpha_1\beta_2}, f_{21}^{\alpha_2\beta_1}, f_{22}^{\alpha_2\beta_2}$ $f_{14}^{\alpha_1\beta_1}, f_{23}^{\alpha_2\beta_2}, f_{13}^{\alpha_1\beta_2}, f_{24}^{\alpha_2\beta_1}$
$P_s(IIB)$	$G_{ij}, \Phi(=0)$	$\Psi_{+i}^{\alpha_1}, \Psi_{-i}^{\alpha_2}$	$(f_{11}^a)^{\alpha_1\beta_1}, (f_{22}^a)^{\alpha_2\beta_2}, (f_{12}^{\alpha_1\beta_2} - f_{21}^{\beta_2\alpha_1})$ $(f_{14}^{\alpha_1\beta_1} - f_{14}^{*\beta_1\alpha_1}), (f_{23}^{\alpha_2\beta_2} - f_{23}^{*\beta_2\alpha_2})$ $(f_{13}^{\alpha_1\beta_2} - f_{24}^{*\beta_2\alpha_1})$
Eff.	$G_{ij}^{eff}, \Phi(=0)$	$(\Psi_{eff})_i^{\alpha_1}$ $(\Psi_{eff})_i^{\alpha_2}$	$(f_{11}^{eff})^{\alpha_1\beta_1}, (f_{12}^{eff})^{\alpha_1\beta_2}, (f_{22}^{eff})^{\alpha_2\beta_2}$ $\bar{f}_{14}^{\alpha_1\beta_1}, \bar{f}_{23}^{\alpha_2\beta_2}, \bar{f}_{13}^{\alpha_1\beta_2}$
Type I	$G_{ij}^I, \Phi^I(=0)$	$(\Psi_I)_i^{\alpha_1}, (\Psi_I)_i^{\alpha_2}$	$(f_{11}^{a(I)})^{\alpha_1\beta_1}, (f_{22}^{a(I)})^{\alpha_2\beta_2}, (f_{12}^{(I)\alpha_1\beta_2} - f_{21}^{(I)\beta_2\alpha_1})$ $(f_{14}^{(I)\alpha_1\beta_1} - f_{14}^{(I)\beta_1\alpha_1*}), (f_{23}^{(I)\alpha_2\beta_2} - f_{23}^{(I)\beta_2\alpha_2*})$ $(f_{13}^{(I)\alpha_1\beta_2} - f_{24}^{(I)\beta_2\alpha_1*})$

Table 3: Superstring theories and their background fields

The last three rows contain the same sets of background fields. Usual identification of type I theory with $P_s(IIB)$ preserves only Ω even fields. We identify type I theory with effective one and obtain improvement with squares of Ω odd type IIB fields.

Consequently, while the Ω even fields appear in known manner, the Ω odd fields, which are eliminated in standard approach, here have two roles. They are source of noncommutativity and their bilinear combinations are additional terms of effective background. So, if naively, type I superstring theory, as Ω even one, can not recognize explicitly Ω odd fields of type IIB theory, it can see them implicitly as Ω even combinations in effective background.

A Gamma matrices and spinors in $D = 10$ and $d = 6$ dimensions

The fermionic variables of the type IIB superstring theory, θ^α and $\bar{\theta}^\alpha$, are 10 dimensional Majorana-Weyl spinors with 16 real components. In order to express them in terms of general complex 32-component Dirac spinors, and to investigate their relation with $D5$ -brane spinors, we have to introduce some representation of gamma matrices. We will use notation and conventions introduced in Appendix B of the first reference in [2]. The gamma matrices in $D = 10$ dimensions are of the form $\Gamma^\mu = (\Gamma^i, \Gamma^a)$

$$\Gamma^i = \gamma^i \otimes \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \quad \Gamma^a = \mathbf{1}_8 \otimes \gamma^a, \quad (\text{A.1})$$

where γ^i ($i = 0, 1, \dots, 5$) is 8×8 representation of gamma matrices in $d = 6$ dimensions, while γ^a ($a = 6, 7, 8, 9$) is 4×4 representation of gamma matrices in $d = 4$ dimensions. Matrices γ^a are of the form

$$\begin{aligned} \gamma^6 &= \begin{pmatrix} -\sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix}, \quad \gamma^7 = \begin{pmatrix} -\sigma_2 & 0 \\ 0 & \sigma_2 \end{pmatrix}, \\ \gamma^8 &= \begin{pmatrix} 0 & \mathbf{1}_2 \\ \mathbf{1}_2 & 0 \end{pmatrix}, \quad \gamma^9 = \begin{pmatrix} 0 & -i\mathbf{1}_2 \\ i\mathbf{1}_2 & 0 \end{pmatrix}, \end{aligned} \quad (\text{A.2})$$

where the matrices σ_i are Pauli 2×2 matrices

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (\text{A.3})$$

For the corresponding γ_{d+1} matrices in $D = 10$ and $d = 6$ dimensions we will use symbols Γ and γ , respectively. According to the definition [2], we have

$$\Gamma \equiv \prod_{\mu=0}^9 \Gamma^\mu = \gamma \otimes \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix}, \quad \gamma \equiv -\prod_{i=0}^5 \gamma^i, \quad (\text{A.4})$$

where Γ and γ are symmetric matrices, $\Gamma^T = \Gamma$ and $\gamma^T = \gamma$.

A.1 Complex conjugation of gamma matrices

The complex conjugation of gamma matrices can be described by operator B_1

$$B_1 = \Gamma^3 \Gamma^5 \dots \Gamma^{d-1}, \quad (\text{A.5})$$

which maps Γ^μ to $\Gamma^{\mu*}$

$$B_1 \Gamma^\mu B_1^{-1} = (-1)^{\frac{d-2}{2}} \Gamma^{\mu*}. \quad (\text{A.6})$$

In the case $D = 10$ operator B_1 is of the form

$$B_1 = \Gamma^3 \Gamma^5 \Gamma^7 \Gamma^9 = b_1 \otimes \begin{pmatrix} 0 & i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix}, \quad (\text{A.7})$$

where b_1 is the corresponding one for $d = 6$

$$b_1 = \gamma^3 \gamma^5. \quad (\text{A.8})$$

Matrix B_1 is symmetric, $B_1^T = B_1$, while the matrix b_1 is antisymmetric, $b_1^T = -b_1$.

A.2 Charge conjugation operator

The transposed gamma matrices satisfy the same algebra as original ones. There are similarity transformations which map from Γ^μ to $-\Gamma^{\mu T}$ and γ^i to $-\gamma^{iT}$

$$C \Gamma^\mu C^{-1} = -\Gamma^{\mu T}, \quad c \gamma^i c^{-1} = -\gamma^{iT}, \quad (\text{A.9})$$

described by charge conjugation operators

$$C = B_1 \Gamma^0, \quad c = b_1 \gamma^0. \quad (\text{A.10})$$

In $D = 10$ dimensions operator C is antisymmetric, $C^T = -C$, while the corresponding one in $d = 6$ dimensions is symmetric, $c^T = c$.

A.3 From ten dimensional to six dimensional spinors

From $D = 10$ Majorana and Weyl conditions

$$\theta^* = B_1 \theta, \quad \Gamma \theta = \theta, \quad (\text{A.11})$$

it follows that independent components of general 32 component Dirac spinor

$$\theta^\alpha = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{pmatrix} = \begin{pmatrix} \theta_1 \\ \theta_2 \\ b_1 \theta_2^* \\ -b_1 \theta_1^* \end{pmatrix}, \quad (\alpha = 1, 2, \dots, 32) \quad (\text{A.12})$$

are two 8 component spinors $\theta_1^{\alpha_1}$ and $\theta_2^{\alpha_2}$ ($\alpha_1, \alpha_2 = 1, 2, \dots, 8$) with constraints

$$\gamma\theta_1 = \theta_1, \quad \gamma\theta_2 = -\theta_2, \quad (\text{A.13})$$

which we recognize as two opposite chirality Weyl spinors in $d = 6$. Similarly, from $\pi^{T*} = \pi^T B_1^T$ and $\pi^T \Gamma^T = \pi^T$, we obtain

$$\pi_\alpha = \begin{pmatrix} \pi_1, & \pi_2, & -\pi_2^* b_1, & \pi_1^* b_1 \end{pmatrix}, \quad (\text{A.14})$$

with

$$\pi_1 \gamma = \pi_1, \quad \pi_2 \gamma = -\pi_2. \quad (\text{A.15})$$

Consequently, we have

$$\pi_\alpha \theta^\alpha = 2\Re(\pi_{\alpha_1} \theta^{\alpha_1} + \pi_{\alpha_2} \theta^{\alpha_2}), \quad (\text{A.16})$$

where symbol \Re means real part of some complex number.

B Field strength $F^{\alpha\beta}$ in terms of antisymmetric tensors

The connection between two descriptions of R-R sector, field strength $F^{\alpha\beta}$ and anti-symmetric tensors $F_{(k)}$ can be established [7], where in short-hand notation $F_{(k)}$ denotes k -rank antisymmetric tensor. It is known that bispinor $F^{\alpha\beta} = S^\alpha (\Gamma^0 \tilde{S})^\beta$, made from same chirality spinors S^α and \tilde{S}^α for type IIB theory, can be expand into complete set of 10 dimensional antisymmetric gamma matrices

$$F^{\alpha\beta} = \sum_{k=0}^D \frac{1}{k!} F_{(k)} \Gamma_{(k)}^{\alpha\beta}, \quad \left[\Gamma_{(k)}^{\alpha\beta} = (C \Gamma^{[\mu_1 \dots \mu_k]})^{\alpha\beta} \right] \quad (\text{B.1})$$

where C is charge conjugation operator defined in (A.10). Here

$$\Gamma^{[\mu_1 \mu_2 \dots \mu_k]} \equiv \Gamma^{\mu_1} \Gamma^{\mu_2} \dots \Gamma^{\mu_k}, \quad (\text{B.2})$$

is completely antisymmetrized product of gamma matrices.

The bispinor $F^{\alpha\beta}$ satisfy chirality condition, $\Gamma F = -F \Gamma$, and, consequently, type IIB theory contains only odd rank tensors $F_{(k)}$. Because of duality relation, the independent tensors are $F_{(1)}$, $F_{(3)}$ and self-dual part of $F_{(5)}$. Using mass-shell condition (massless Dirac equation for $F^{\alpha\beta}$) these tensors can be solved in terms of potentials $F_{(k)} = dA_{(k-1)}$, so that IIB theory contains the potentials $A_{(0)}$, $A_{(2)}$ and $A_{(4)}$. The number of independent components of $F^{\alpha\beta}$ is exactly 256, while the number of degrees of freedom is 64.

The matrices $\Gamma_{(1)}$ and $\Gamma_{(5)}$ are symmetric in spinor indices, while the matrix $\Gamma_{(3)}$ is antisymmetric. So, the symmetric part of $F^{\alpha\beta}$, $F_s^{\alpha\beta} = \frac{1}{2}(F^{\alpha\beta} + F^{\beta\alpha})$, corresponds to the field strengths $F_{(1)}$ and $F_{(5)}$, and antisymmetric part, $F_a^{\alpha\beta} = \frac{1}{2}(F^{\alpha\beta} - F^{\beta\alpha})$, corresponds to the field strength $F_{(3)}$.

Using the form of bispinor $F^{\alpha\beta} = S^\alpha(\Gamma^0 \tilde{S})^\beta$, the form of spinors (A.12) and expression for Γ^0 (A.1), we obtain

$$F = \begin{pmatrix} f_{11} & -f_{12} & f_{13}b_1 & f_{14}b_1 \\ f_{21} & -f_{22} & f_{23}b_1 & f_{24}b_1 \\ b_1 f_{24}^* & -b_1 f_{23}^* & b_1 f_{22}^* b_1 & b_1 f_{21}^* b_1 \\ -b_1 f_{14}^* & b_1 f_{13}^* & -b_1 f_{12}^* b_1 & -b_1 f_{11}^* b_1 \end{pmatrix}, \quad (\text{B.3})$$

where

$$f_{11} = S_1(\gamma^0 \tilde{S}_1), \quad f_{12} = S_1(\gamma^0 \tilde{S}_2), \quad f_{21} = S_2(\gamma^0 \tilde{S}_1), \quad f_{22} = S_2(\gamma^0 \tilde{S}_2), \quad (\text{B.4})$$

$$f_{13} = S_1(\gamma^0 \tilde{S}_2^*), \quad f_{14} = S_1(\gamma^0 \tilde{S}_1^*), \quad f_{23} = S_2(\gamma^0 \tilde{S}_2^*), \quad f_{24} = S_2(\gamma^0 \tilde{S}_1^*), \quad (\text{B.5})$$

and f_{11} corresponds to $f^{\alpha_1\beta_1}$, f_{12} to $f^{\alpha_1\beta_2}$ etc.

The chirality condition $\Gamma F = -F\Gamma$ splits into eight conditions

$$\gamma f_{11} = -f_{11}\gamma, \quad \gamma f_{12} = f_{12}\gamma, \quad \gamma f_{21} = f_{21}\gamma, \quad \gamma f_{22} = -f_{22}\gamma, \quad (\text{B.6})$$

$$\gamma f_{13} = f_{13}\gamma, \quad \gamma f_{14} = -f_{14}\gamma, \quad \gamma f_{23} = -f_{23}\gamma, \quad \gamma f_{24} = f_{24}\gamma. \quad (\text{B.7})$$

We are going to apply the same procedure as in $D = 10$ and expand f_{11} , f_{12} , f_{21} , f_{22} , f_{13} , f_{14} , f_{23} and f_{24} into complete set of 6 dimensional antisymmetric gamma matrices $\gamma_{(k)} \equiv (c\gamma^{[i_1 \dots i_k]})$. From the chirality conditions for f_{11} , f_{22} , f_{14} and f_{23} it follows that they contain odd rank tensors $f_{(1)}$ and self-dual part of $f_{(3)}$, while from the chirality conditions for f_{12} , f_{21} , f_{13} and f_{24} it follows that they contain even rank tensors $f_{(0)}$ and $f_{(2)}$. Using mass-shell condition these tensors can be expressed in terms of potentials as $f_{(k)} = da_{(k-1)}$. Consequently, f_{11} , f_{22} , f_{14} and f_{23} contain potentials $a_{(0)}$ and self-dual part of $a_{(2)}$, while f_{12} , f_{21} , f_{13} and f_{24} contain potential $a_{(1)}$ ($a_{(-1)}$ is not a physical degree of freedom). The number of independent components of f_{11} , f_{22} , f_{14} , f_{23} , f_{12} , f_{21} , f_{13} and f_{24} is 128, while the number of degrees of freedom is 32.

In analogy with the case $D = 10$, in $d = 6$ dimensions, symmetric parts of f_{11} , f_{22} , f_{14} and f_{23} correspond to the field strength $f_{(3)}$, while their antisymmetric parts correspond to the field strength $f_{(1)}$.

Using the expressions (A.14) and (B.3), we have

$$\begin{aligned} \pi_{\alpha} F^{\alpha\beta} \bar{\pi}_{\beta} &= \\ &= 2\Re(\pi_{\alpha_1} f_{11}^{\alpha_1\beta_1} \bar{\pi}_{\beta_1} + \pi_{\alpha_1} f_{14}^{\alpha_1\beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_2} f_{22}^{\alpha_2\beta_2} \bar{\pi}_{\beta_2} - \pi_{\alpha_2} f_{23}^{\alpha_2\beta_2} \bar{\pi}_{\beta_2}^*) \\ &+ 2\Re(\pi_{\alpha_2} f_{21}^{\alpha_2\beta_1} \bar{\pi}_{\beta_1} - \pi_{\alpha_1} f_{12}^{\alpha_1\beta_2} \bar{\pi}_{\beta_2} + \pi_{\alpha_2} f_{24}^{\alpha_2\beta_1} \bar{\pi}_{\beta_1}^* - \pi_{\alpha_1} f_{13}^{\alpha_1\beta_2} \bar{\pi}_{\beta_2}^*). \end{aligned} \quad (\text{B.8})$$

C Complex coordinates and their canonically conjugated momenta

Lagrangian contains complex coordinates and momenta. It is important to determine the basic Poisson structure between coordinates and momenta.

Let us start with one term from the Lagrangian (2.11)

$$2\Re[-\pi_{\alpha_1}(\partial_\tau - \partial_\sigma)\theta^{\alpha_1}] . \quad (\text{C.1})$$

Decomposing θ^{α_1} and π_{α_1} in real and imaginary parts

$$\theta^{\alpha_1} = \theta_1^{\alpha_1} + i\theta_2^{\alpha_1}, \quad \pi_{\alpha_1} = \pi_{\alpha_1}^1 + i\pi_{\alpha_1}^2, \quad (\text{C.2})$$

expression (C.1) gets the form

$$-2\pi_{\alpha_1}^1(\partial_\tau - \partial_\sigma)\theta_1^{\alpha_1} + 2\pi_{\alpha_1}^2(\partial_\tau - \partial_\sigma)\theta_2^{\alpha_1}. \quad (\text{C.3})$$

The canonically conjugated momentum for $\theta_1^{\alpha_1}$ is $2\pi_{\alpha_1}^1$, while for $\theta_2^{\alpha_1}$ is $-2\pi_{\alpha_1}^2$, which means that all nonzero Poisson brackets are

$$\{\theta_1^{\alpha_1}(\sigma), \pi_{\beta_1}^1(\bar{\sigma})\} = -\frac{1}{2}\delta^{\alpha_1}_{\beta_1}\delta(\sigma - \bar{\sigma}), \quad \{\theta_2^{\alpha_1}(\sigma), \pi_{\beta_1}^2(\bar{\sigma})\} = \frac{1}{2}\delta^{\alpha_1}_{\beta_1}\delta(\sigma - \bar{\sigma}). \quad (\text{C.4})$$

Using these relations we easily obtain Poisson brackets of the complex variables

$$\{\theta^{\alpha_1}(\sigma), \pi_{\beta_1}(\bar{\sigma})\} = -\delta^{\alpha_1}_{\beta_1}\delta(\sigma - \bar{\sigma}), \quad \{\theta^{\alpha_1}(\sigma), \pi_{\beta_1}^*(\bar{\sigma})\} = 0, \quad \{\theta^{*\alpha_1}(\sigma), \pi_{\beta_1}(\bar{\sigma})\} = 0, \quad (\text{C.5})$$

where $*$ means complex conjugation. This means that θ^{α_1} and π_{α_1} are canonically conjugated complex variables, while θ^{α_1} and $\pi_{\beta_1}^*$ are canonically independent. The same procedure can be repeated for all other terms in Lagrangian.

Similarly, in definition of canonical Hamiltonian we have

$$\dot{\theta}_1^{\alpha_1}2\pi_{\alpha_1}^1 - \dot{\theta}_2^{\alpha_2}2\pi_{\alpha_2}^2 = 2\Re(\dot{\theta}^{\alpha_1}\pi_{\alpha_1}) = 2\Re(\dot{\theta}^{*\alpha_1}\pi_{\alpha_1}^*). \quad (\text{C.6})$$

D Consistency procedure for fermionic constraints - explicit expressions

In order to investigate the consistency of the fermionic constraints (3.15) and (3.16), we have to apply consistency procedure to the variables

$$A^{(0)} = (\theta^{\alpha_1}, \theta^{\alpha_2}, \bar{\theta}^{\alpha_1}, \bar{\theta}^{\alpha_2}, \pi_{\alpha_1}, \pi_{\alpha_2}, \bar{\pi}_{\alpha_1}, \bar{\pi}_{\alpha_2}).$$

If we define the recurrent equation

$$A^{(n+1)} \equiv \{H_c, A^{(n)}\}, \quad (n \geq 0) \quad (\text{D.1})$$

we obtain

$$(\Theta^{\alpha_1})^{(2n)} = \partial_\sigma^{(2n)} \theta^{\alpha_1}, \quad (\Theta^{\alpha_2})^{(2n)} = \partial_\sigma^{(2n)} \theta^{\alpha_2}, \quad (\text{D.2})$$

$$\begin{aligned} (\Theta^{\alpha_1})^{(2n+1)} &= -\partial_\sigma^{(2n+1)} \theta^{\alpha_1} - \Psi_i^{\alpha_1} \partial_\sigma^{(2n+1)} x^i - \frac{1}{2\kappa} f_{11}^{\alpha_1 \beta_1} \partial_\sigma^{(2n)} \bar{\pi}_{\beta_1} - \frac{1}{2\kappa} f_{14}^{\alpha_1 \beta_1} \partial_\sigma^{(2n)} \bar{\pi}_{\beta_1}^* \\ &+ \frac{1}{2\kappa} f_{12}^{\alpha_1 \beta_2} \partial_\sigma^{(2n)} \bar{\pi}_{\beta_2} + \frac{1}{2\kappa} f_{13}^{\alpha_1 \beta_2} \partial_\sigma^{(2n)} \bar{\pi}_{\beta_2}^* + \frac{1}{2\kappa} G^{ij} \partial_\sigma^{(2n)} (I_{+i} + I_{-i}) \Psi_j^{\alpha_1}, \\ (\Theta^{\alpha_2})^{(2n+1)} &= -\partial_\sigma^{(2n+1)} \theta^{\alpha_2} - \Psi_i^{\alpha_2} \partial_\sigma^{(2n+1)} x^i + \frac{1}{2\kappa} f_{22}^{\alpha_2 \beta_2} \partial_\sigma^{(2n)} \bar{\pi}_{\beta_2} + \frac{1}{2\kappa} f_{23}^{\alpha_2 \beta_2} \partial_\sigma^{(2n)} \bar{\pi}_{\beta_2}^* \\ &- \frac{1}{2\kappa} f_{21}^{\alpha_2 \beta_1} \partial_\sigma^{(2n)} \bar{\pi}_{\beta_1} - \frac{1}{2\kappa} f_{24}^{\alpha_2 \beta_1} \partial_\sigma^{(2n)} \bar{\pi}_{\beta_1}^* + \frac{1}{2\kappa} G^{ij} \partial_\sigma^{(2n)} (I_{+i} + I_{-i}) \Psi_j^{\alpha_2}. \end{aligned} \quad (\text{D.3})$$

$$(\bar{\Theta}^{\alpha_1})^{(2n)} = \partial_\sigma^{(2n)} \bar{\theta}^{\alpha_1}, \quad (\bar{\Theta}^{\alpha_2})^{(2n)} = \partial_\sigma^{(2n)} \bar{\theta}^{\alpha_2}, \quad (\text{D.4})$$

$$\begin{aligned} (\bar{\Theta}^{\alpha_1})^{(2n+1)} &= \partial_\sigma^{(2n+1)} \bar{\theta}^{\alpha_1} + \bar{\Psi}_i^{\alpha_1} \partial_\sigma^{(2n+1)} x^i + \frac{1}{2\kappa} \partial_\sigma^{(2n)} \pi_{\beta_1} f_{11}^{\beta_1 \alpha_1} + \frac{1}{2\kappa} \partial_\sigma^{(2n)} \pi_{\beta_1}^* f_{14}^{* \beta_1 \alpha_1} \\ &+ \frac{1}{2\kappa} f_{21}^{\beta_2 \alpha_1} \partial_\sigma^{(2n)} \pi_{\beta_2} + \frac{1}{2\kappa} f_{24}^{* \beta_2 \alpha_1} \pi_{\beta_2}^* + \frac{1}{2\kappa} G^{ij} \partial_\sigma^{(2n)} (I_{+i} + I_{-i}) \bar{\Psi}_j^{\alpha_1}, \\ (\bar{\Theta}^{\alpha_2})^{(2n+1)} &= \partial_\sigma^{(2n+1)} \bar{\theta}^{\alpha_2} + \bar{\Psi}_i^{\alpha_2} \partial_\sigma^{(2n+1)} x^i - \frac{1}{2\kappa} \partial_\sigma^{(2n)} \pi_{\beta_2} f_{22}^{\beta_2 \alpha_2} - \frac{1}{2\kappa} \partial_\sigma^{(2n)} \pi_{\beta_2}^* f_{23}^{* \beta_2 \alpha_2} \\ &- \frac{1}{2\kappa} f_{12}^{\beta_1 \alpha_2} \partial_\sigma^{(2n)} \pi_{\beta_1} - \frac{1}{2\kappa} f_{13}^{* \beta_1 \alpha_2} \partial_\sigma^{(2n)} \pi_{\beta_1}^* + \frac{1}{2\kappa} G^{ij} \partial_\sigma^{(2n)} (I_{+i} + I_{-i}) \bar{\Psi}_j^{\alpha_2}. \end{aligned} \quad (\text{D.5})$$

$$\begin{aligned} (\Pi_{\alpha_1})^{(n)} &= (-1)^n \partial_\sigma^{(n)} \pi_{\alpha_1}, \quad (\Pi_{\alpha_2})^{(n)} = (-1)^n \partial_\sigma^{(n)} \pi_{\alpha_2}, \\ (\bar{\Pi}_{\alpha_1})^{(n)} &= \partial_\sigma^{(n)} \bar{\pi}_{\alpha_1}, \quad (\bar{\Pi}_{\alpha_2})^{(n)} = \partial_\sigma^{(n)} \bar{\pi}_{\alpha_2}. \end{aligned} \quad (\text{D.6})$$

Defining the function

$$\mathcal{A}(\sigma) \equiv \sum_{n=0}^{\infty} \frac{\sigma^n}{n!} A^{(n)}(\sigma=0), \quad (\text{D.7})$$

we introduce compact σ dependent expressions for the corresponding variables after consistency procedure

$$\begin{aligned} \Theta^{\alpha_1}(\sigma) &= \theta^{\alpha_1}(-\sigma) - \Psi_i^{\alpha_1} \tilde{q}^i(\sigma) - \frac{1}{2\kappa} f_{11}^{\alpha_1 \beta_1} \int_0^\sigma d\sigma_1 P_s \bar{\pi}_{\beta_1} - \frac{1}{2\kappa} f_{14}^{\alpha_1 \beta_1} \int_0^\sigma d\sigma_1 P_s \bar{\pi}_{\beta_1}^* \\ &+ \frac{1}{2\kappa} f_{12}^{\alpha_1 \beta_2} \int_0^\sigma d\sigma_1 P_s \bar{\pi}_{\beta_2} + \frac{1}{2\kappa} f_{13}^{\alpha_1 \beta_2} \int_0^\sigma d\sigma_1 P_s \bar{\pi}_{\beta_2}^* + \frac{1}{2\kappa} G^{ij} \Psi_i^{\alpha_1} \int_0^\sigma d\sigma_1 P_s (I_{+j} + I_{-j}), \\ \Theta^{\alpha_2}(\sigma) &= \theta^{\alpha_2}(-\sigma) - \Psi_i^{\alpha_2} \tilde{q}^i(\sigma) + \frac{1}{2\kappa} f_{22}^{\alpha_2 \beta_2} \int_0^\sigma d\sigma_1 P_s \bar{\pi}_{\beta_2} + \frac{1}{2\kappa} f_{23}^{\alpha_2 \beta_2} \int_0^\sigma d\sigma_1 P_s \bar{\pi}_{\beta_2}^* \\ &- \frac{1}{2\kappa} f_{21}^{\alpha_2 \beta_1} \int_0^\sigma d\sigma_1 P_s \bar{\pi}_{\beta_1} - \frac{1}{2\kappa} f_{24}^{\alpha_2 \beta_1} \int_0^\sigma d\sigma_1 P_s \bar{\pi}_{\beta_1}^* \\ &+ \frac{1}{2\kappa} G^{ij} \Psi_i^{\alpha_2} \int_0^\sigma d\sigma_1 P_s (I_{+j} + I_{-j}), \end{aligned} \quad (\text{D.8})$$

$$\begin{aligned}
\bar{\Theta}^{\alpha_1}(\sigma) &= \bar{\theta}^{\alpha_1}(\sigma) + \bar{\Psi}_i^{\alpha_1} \tilde{q}^i(\sigma) + \frac{1}{2\kappa} f_{11}^{\beta_1 \alpha_1} \int_0^\sigma d\sigma_1 P_s \pi_{\beta_1} + \frac{1}{2\kappa} f_{14}^{*\beta_1 \alpha_1} \int_0^\sigma d\sigma_1 P_s \pi_{\beta_1}^* \\
&+ \frac{1}{2\kappa} f_{21}^{\beta_2 \alpha_1} \int_0^\sigma d\sigma_1 P_s \pi_{\beta_2} + \frac{1}{2\kappa} f_{24}^{*\beta_2 \alpha_1} \int_0^\sigma d\sigma_1 P_s \pi_{\beta_2}^* + \frac{1}{2\kappa} G^{ij} \bar{\Psi}_i^{\alpha_1} \int_0^\sigma d\sigma_1 P_s (I_{+j} + I_{-j}), \\
\bar{\Theta}^{\alpha_2}(\sigma) &= \bar{\theta}^{\alpha_2}(\sigma) + \bar{\Psi}_i^{\alpha_2} \tilde{q}^i(\sigma) - \frac{1}{2\kappa} f_{22}^{\beta_2 \alpha_2} \int_0^\sigma d\sigma_1 P_s \pi_{\beta_2} - \frac{1}{2\kappa} f_{23}^{*\beta_2 \alpha_2} \int_0^\sigma d\sigma_1 P_s \pi_{\beta_2}^* \\
&- \frac{1}{2\kappa} f_{12}^{\beta_1 \alpha_2} \int_0^\sigma d\sigma_1 P_s \pi_{\beta_1} - \frac{1}{2\kappa} f_{13}^{*\beta_1 \alpha_2} \int_0^\sigma d\sigma_1 P_s \pi_{\beta_1}^* \\
&+ \frac{1}{2\kappa} G^{ij} \bar{\Psi}_i^{\alpha_2} \int_0^\sigma d\sigma_1 P_s (I_{+j} + I_{-j}), \tag{D.9}
\end{aligned}$$

$$\begin{aligned}
\Pi_{\alpha_1}(\sigma) &= \pi_{\alpha_1}(-\sigma), \quad \Pi_{\alpha_2}(\sigma) = \pi_{\alpha_2}(-\sigma), \\
\bar{\Pi}_{\alpha_1}(\sigma) &= \bar{\pi}_{\alpha_1}(\sigma), \quad \bar{\Pi}_{\alpha_2}(\sigma) = \bar{\pi}_{\alpha_2}(\sigma). \tag{D.10}
\end{aligned}$$

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